

## Article

# Tourism development: A capacity dynamic

**Bruno Marques**MEMIAD, Université des Antilles, Pôle Martinique, Campus de Schoelcher, BP 7004, Schoelcher 97233, France; [b.marques@orange.fr](mailto:b.marques@orange.fr)**CITATION**

Marques B. Tourism development: A capacity dynamic. *Smart Tourism*. 2024; 5(1): 2491.  
<https://doi.org/10.54517/st.v5i1.2491>

**ARTICLE INFO**

Received: 16 January 2024  
Accepted: 29 January 2024  
Available online: 21 February 2024

**COPYRIGHT**

Copyright © 2024 by author(s).  
*Smart Tourism* is published by Asia Pacific Academy of Science Pte. Ltd. This work is licensed under the Creative Commons Attribution (CC BY) license.  
<https://creativecommons.org/licenses/by/4.0/>

**Abstract:** The internationally adopted definition of tourism prompts the development of a systemic, dynamic approach to tourism development. The paper proposes to conceptualize tourism development as a system interlinking three agents: transport, domestic tourism activities, and the visitor, generating three types of tourism development dynamics. In a second step, it uses this framework to develop, with a minimalist set of hypotheses, a capacity-based model that enables it to consider destination tourism development as a microfounded supply-driven systemic dynamic process. Through the lens of the model, exhaustion or an asymmetric distribution of market power may halt destination tourism development. Using the model's framework, the structuring forces of the Tourism Area Life Cycle (TALC) are explained by the dual impact of capacity dynamics: accelerating by increasing arrivals and, at the same time, decelerating by declining price elasticities.

**Keywords:** tourism development; systemic dynamic approach; capacity dynamics; microfoundation; TALC

## 1. Introduction: A systemic dynamic approach of destination tourism development

Visitors, activities, and travel are the three pillars that underpin the definition of tourism adopted internationally in 2010 (United Nations, World Tourism Organization, Organization for Economic Co-operation and Development, and Eurostat [1] consider that “Tourism is a social, cultural, and economic phenomenon that involves the movement of people). This definition has two consequences, or corollaries, and allows us to conceptualize destination tourism development through a dynamic systemic approach.

### 1.1. Two corollaries of international definition of tourism

The first corollary is related to the notion of tourism development. By installing visitors at the core of the tourism phenomenon, the international definition of tourism enshrines the time evolution of tourist attendance (the paper uses indifferently tourism flow, arrivals, attendance, or tourist numbers to refer to the number of tourists arriving at a destination) as the primary indicator for measuring the tourism development of a destination. All the other indicators of tourism development (bed nights, receipts, etc.) are in fact or can be correlated with tourist numbers. This reading of tourism development through time is rooted in a long-term perspective analysis of the change process in tourism, as comprehensively exposed by Noreen Maree Breakey's thesis [2]. Butler's [3] tourism area life cycle (TALC) remains the most widely used long-term model of tourism development in this field of research. It differs from the structural approach extensively adopted by the economics of tourism, which focuses on identifying the determinants of tourism demand, once assimilated to the tourism

flow, from a perspective where time is not the primary factor. ( The structural approach can be found in Lim [4], and Stabler et al. [5] carry out a comprehensive literature review of tourism demand modeling, which distinguishes two fields of modeling. The first focuses on the identification of the factors determining tourism flows, initially based on theoretical models, mainly gravity models inspired by international trade theories, then via empirical approaches presented in Li and al. [6], Sinclair and Stabler [7], and Stabler et al. [5]. The second, based on a more grounded theoretical base: static and temporal microeconomic theory of demand, The almost ideal demand system model of Deaton and Muellbauer [8], the characteristics approach of Lancaster [9], and the discrete choice models of Anas [10], Morley [11], and Alegre and Pou [12] provide a set of explanatory factors for tourism expenditure. Song et al. [13,14] provide a comprehensive review of the theoretical, empirical, and methodological literature on the identification of the determinants of tourism attendance. In the latest Handbook of Tourism Economics, Divisekera [15] presents the most recent theoretical and empirical work resulting from this approach, which also integrates structural changes, seasonality, and exogenous events.

The second consequence of the conceptual definition of tourism is the intrinsic link between tourism and transport, since traveling is a component of the tourism phenomenon. According to the international definition, there is no tourism without visitors, but there is also no tourism without travel and thus without transport. References abound to the interdependence of tourism and transport. Macintosh et al. [16] define tourism (including transport) as a coordinated system. Prideaux [17] and Lumsdon and Page [18] conclude that transport and tourism are structurally linked by an asymmetric relationship: the latter's demand and revenues are set by the former through infrastructure and carrier decisions. Leaving aside the search for unidirectional causality, Gay [19] indicates that the links between tourism and transport are "cumulative" and that "we must not fall into the "mediological" trap... that would make tourism and tourists elements determined by the media, i.e., transport". The concepts of "tourist transport" and "supply chain" [20] recognize the intrinsically systemic dimension of the relationship between tourism and transport. For Lohmann and Duval [21], the link between transport and tourism is "symbiotic" and a matter of "co-dependence". Finally, through the concept of connectivity, the World Tourism Organization (UNWTO) [22] underlines the importance of transport capacity dynamics when referring to the imbalances between transport and tourism suppliers.

The definition itself and its corollaries offer a framework to comprehend the development of tourism at a destination as a dynamic system.

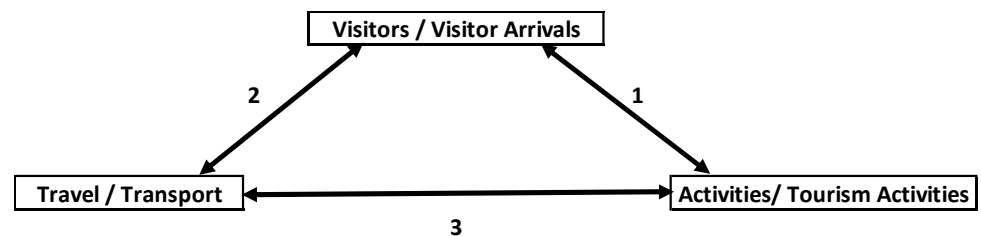
## **1.2. A systemic dynamic approach of destination tourism development**

From the standpoint of a destination, a tourism development phenomenon is possible if three components are gathered (the pillars of the international definition of tourism): visitors, tourism activities, and transport. These three elements constitute a dynamic system because they are linked and influence each other at one point in time and during that time. Following Durand [23], the systemic approach defines a system as a finite set of elements linked by linear or non-linear interactions, more or less complex: bidirectional and/or circular, which determine the dynamic evolution of this

set as described by one or more indicators. The systemic approach considers that the elements of the system are not necessarily aware that they belong to the system, which differentiates it from game theory. As such, the systemic approach facilitates the study of non-cooperative phenomena. The tourism dynamic system can also be read as a dynamic economic system, composed of three elements organized around two poles:

- demand: visitors;
- Supply: activities/tourism activities and travel/transport are the supply sides of the system, which produce the goods and services provided to tourists (activities are not necessarily tourism activities *per se*). They become tourism activities through tourist consumption, and the World Tourism Organization discriminates between “tourism characteristic products” and “tourism connected products” for the analysis of the economic impact of tourism.

**Figure 1** figures out the destination tourism development system and is a symbolic representation of the systemic, dynamic approach to destination tourism development.



To avoid repetition, the paper uses indifferently destination tourism development, destination tourism development system, or tourism development system. As mentioned earlier, a synthetic indicator or a holistic view of the state of the system at one point in time is visitor attendance. Attendance is preferred to the overall tourism revenue of the two sectors since the latter is dependent on the former. It was also preferred to capacity since there is no tourism without tourists, even if capacity exists. Sectoral profits and occupation coefficients are also dependent on attendance. Its evolution over time indicates the tourism development of the destination. The links between the elements of the tourism development system, the bi-directional arrows, express their bi-causal dynamic influences. These links govern the state and motion of the development tourism system and, therefore, its holistic indicator: tourism arrivals. They cause and plot the evolution of tourist attendance over time. Each bi-causal link expresses a sub-dynamic enacted by the relations between a pair of components and identifies a type of destination tourism development dynamic. As a synthesis, a systemic dynamic approach conceives destination tourism development as a system composed of the three components of the international definition of tourism, with links between each pair of the components expressing a specific sub-dynamic enabling, through their combination, the dynamic of the whole system.

Bi-directional arrow n°1 expresses a “by activities/products” tourism development, where qualitative and quantitative demand-supply dynamics link visitors and the destination itself. On the qualitative side, the types of tourists (demographics, tastes, preferences, and expectations) influence the different activities produced by the destinations. Inversely, the features of the destination (physical nature (sea, mountains, towns, scenery, etc.), culture, and local productions (types of

restaurants, accommodations, leisure)) select the type of tourists. From a quantitative perspective, this dynamic reflects the demand-supply process that causes tourism development. On the demand side, tourist arrivals are the impetus to create tourism products or for the gradual expansion of the consumption basket of tourists during their stay in the destination. On the supply side, the innovation process generates new activities for new types of visitors and, therefore, tourism arrivals. This tourism development dynamic can be related to the works of Candela et al. [24,25] and Andergassen et al. [26].

Arrow n°2 is a “transport-based” destination tourism development dynamic, which is also a crossed-cause demand-supply dynamic. It refers to technological effects on destination tourism development due to the increasing capacity of transport and reduced travel time. These supply-side innovations entail new transport means and expand the markets open to destinations. Conversely, those innovations are responses to the will of new territories to develop tourism as a consequence of the expansion of tourism demand due to economic growth. This dynamic can be related to the aforementioned literature [Macintosh et al. [16], Prideaux [17], Lumsdon and Page [18], Gay [19], Page [20], Lohmann and Duval [21], and some empirical studies, where the price of transport is an explanatory factor of tourist demand [5].

Unlike arrows n°1 and n°2, arrow n°3 is a full “supply side sub-dynamic”, for it does not directly relate the demand and supply sides of the tourism development system. It reflects the way in which the interrelationship between the service providers (the supply pole of the system), conceptually pivotal to the definition of tourism, is able to generate tourist attendance and thus tourism development. The rationale behind this dynamic can be summarized or comprehended as follows: An insufficient transport supply hinders the development of the number of tourists. Simultaneously, limited local tourism activities discourage the development of transport. According to our best knowledge, destination tourism development through the transport-activity dynamic has not yet been examined.

Similarly, we don't have any knowledge of analyses of destination tourism development, i.e., the long-term change process of destination tourism, combining the three systemic sub-dynamics inspired by the previously exposed systemic dynamic approach to destination tourism development.

The paper proposed a formalized model of the systemic dynamic approach of Destination Tourism Development previously exposed, which allows to combine simultaneously its three sub-dynamics. It explores the mechanics of destination tourism development through a system of dynamic equations. The model enables us to better characterize destination tourism development. As such, it can provide tourism planners with tools to manage the systemic interactions between the components during destination tourism development. It is general enough to provide a microfounded understanding of the TALC.

Three sections organize the paper. The first section presents a formalized systemic dynamic model of destination tourism development and the learnings it brings to characterize destination tourism development. In a second section, the model offers a frame to microfound the TALC. Some concluding remarks end up in the paper.

## **2. A systemic dynamic model of destination tourism development**

This section presents a formalized systemic dynamic model of destination tourism development, combining its three sub-dynamics, inspired by the previous approach. A first sub-section exposed the model mechanics based on a minimalist set of hypotheses. A second sub-section outlines the understanding of destination tourism development the model can provide.

### **2.1. A capacity-based destination tourism development model**

This sub-section presents successively: the hypotheses of the model, the agents involved in the destination tourism development system, their behavior, their inter-links, and the effects of these on tourism development.

Assumptions: The formalized version of the systemic dynamic destination tourism development approach adopts four assumptions that bring it closer to tourism reality:

- Destinations are different and cannot be perfect substitutes, inducing that transport (giving access to them) and tourism activities (in the destination) considered as economic sectors can be modeled as price makers maximizing revenue monopolist firms (hereafter transport and tourism activities sectors are referred to as sector- firms);
- Sector firms supply capacity units of transport and reception measurable in numbers of persons, combined as complementary goods bought by optimizing visitors;
- Sector-firm production technology depends uniquely on capital (due to the acknowledged complementary relationship between capacity and the labor factor in transport and tourism activities), and their investment function adopts the internal financial theory [27–29]. This is to be coherent with their maximizing revenue behavior [30] and because a perfect financial market does not necessarily exist at the destination level, especially for small tourism activities.
- A non-cooperative, incomplete, and imperfect information frame for the relations between the two sector firms.

#### **2.1.1. Agents and behaviors**

The supply pole of the tourism development system consists of two production agents: the economic sectors of transport and tourism activities and products. The transport sector aggregates all the firms that serve the destination. It offers a transport service, i.e., the possibility for a visitor to reach the destination. The sector produces units of transport capacity, and its overall production is equal to the overall transport capacity available for the destination in a given period. It is measured by the maximum number of people that can be transported. The tourism activities sector is the aggregation of all firms that offer complementary and substitutable services to tourist visitors (accommodation, catering, and leisure activities). The various tourism services share a common feature: their limited receiving capacity, or the maximum number of people that can be received in a given period. Thus, the tourism activities sector offers a reception capacity at the destination. It produces units of reception capacity linked to a given spacetime. Its overall production is the receiving capacity during a given time interval, measured by the maximum number of people. This approach to the

tourism sector makes it possible to consider all types of visitors: tourists and excursionists. Thus, the tourism sector reflects a variety of situations, from hotel establishments to attractions and parks up to the destination itself in its space dimension: spatial, social, and environmental (carrying capacity). The model considers the two sectors as monopolistic; henceforth, it is called the sector firm in the paper. The monopolistic feature of the two sectors expresses the imperfect nature of the tourism market (close to monopolistic competition), where the various destinations (transport and tourism sectors) are not perfectly substitutable, as Bull [31] suggests. As monopolistic firms, transport and tourism activities are “price-makers”. They set the price of their product, the unit of transport and reception capacity, under the sole constraint of their overall capacity and considering the only demand for their own product, without any consideration of possible interactions. Thus, their behavior indicates a limited awareness of system interactions (incomplete information), expressing specific knowledge or a set of empirical beliefs about the demand for their own product. Therefore, the model reflects the actual non-coordination situation that generally prevails in tourism activities, on the one hand between the two sector firms and, on the other hand, between the multitude of branches that tourism activities aggregate. On the supply side, the tourism development system models a “non-cooperative, incomplete, and imperfect information” situation.

Tourist visitors are consumers of transport and reception capacity units. To be in a touring situation, visitors must necessarily buy the two services, hence their complementarity. Therefore, the number of tourism activities (transport bundles or packages) equals the number of tourists, i.e., the number of visitors transported and received. This equivalence results from the strict correspondence between a bundle of transport-tourism goods and a visitor. Thus, the number of bundles of the two complementary goods is similar to the tourist flow since their combination corresponds to a transported and a received visitor number. The complementarity of the transport and tourism units ensures a strict equivalence of their respective demands with the tourist flow. As “price takers”, under the constraint of their travel budget (tourism + transport), tourists maximize a utility function that integrates transport and tourism goods as defined above (capacity of transport and reception unit). The visitor’s optimization behavior determines the number of tourists visiting the destination.

The previous presentation of the different agents allowed us to formalize their behavior as follows: Index “tr” is for transport and “to” for tourism;  $i$  indicates indifferently the sector firms:

The behaviors of sector-firms: The decision variable of the two monopolist sector-firms is the price of their product: the unit of reception or transport capacity. They maximize their revenue: the product of price ( $p_i$ ) by their perception or knowledge of the demand for their service  $[Q_i(p_i) \text{ with } \frac{\delta Q_i}{\delta p_i} < 0]$ , under the constraint of a fixed overall capacity = sum of the capacity units produced and available in a given period ( $T_i$ ). The sector-firms optimization program is as follows:  $\text{Max} p_i Q_i(p_i)$  under constraint,  $T_i \geq Q_i(p_i)$ . The solution of the program (see Appendix A) gives the price function of the sector-firms:

$$p_i^* = Q_i^{-1}(T_i) \quad (1)$$

where  $Q_i^{-1}$  is the inverse function of  $Q_i$ . Prices are inversely related to capacities;

increase (decrease) in supply entails decrease (increase) in price.  $p_i^*$  are short term prices, with their time motion in the long run set by capacities time motion.

According to the 4th assumption sector-firms' outputs are  $T_i = G_i(K_i)$  with  $K_i$  the sector-firm capital, it comes  $K = G_i^{-1}(T_i)$ , with  $G_i' > 0$  and  $G_i(0) = 0$  and  $\dot{K}_i$  their investment function depends on profit and/or capacity utilization (or occupancy) rates. Defining  $F$  as tourist arrivals and:

- Sector-firm profits as:  $\pi_i = Q_i^{-1}(T_i)F - d_i[G_i^{-1}(T_i)]$ ; with  $d_i[K = G_i^{-1}(T_i)]$ , a cost function describing expenses linked to the use of capital  $K_i$  (maintenance, rents, various costs related to sustainability, Interest rate in case of debt financing);
- Capacity occupancy rates as:  $\psi_i = \frac{F}{T_i}$ .

it comes  $\dot{K}_i = h_i(\pi_i, \psi_i) = h_i(F, T_i)$  (with  $h_i' > 0$  and  $h_i(0) = 0$ , and  $\dot{K}_i$  can be  $< 0$ ) and  $\dot{T}_i = \left(\frac{\delta G_i^{-1}}{\delta T_i}\right)^{-1} h_i(F, T_i)$  as  $\frac{\delta G_i^{-1}}{\delta T_i} \dot{T}_i = \dot{K}_i \cdot h_i$  (with both or one of the arguments:  $\pi$  and/or  $\psi$ ) can be thought as a financing function or alternatively as performance function: a combination of internal performance indicators that triggers investment. Each equation of system Equation (2) below, relates capacity dynamic of each sector-firms to tourism attendance and the level of its own capacity.

$$\dot{T}_i = \left(\frac{\delta G_i^{-1}}{\delta T_i}\right)^{-1} h_i(F, T_i) \quad (2)$$

At this stage, the system  $\dot{T}_i$  is not a reaction functions system because each equation does not integrate the capacities or conjectured actions, of the other sector-firm.

The behavior of tourist visitors: Under the constraint of their expenditure whose prices come from the behavior of the sector-firms:  $R = p_{tr}^* Q_{tr} + p_{to}^* Q_{to}$ , ( $Q_i$  is the quantity consumed of transport and tourism goods defined as units of transport and reception capacity for a given period), tourist visitors maximize a utility function with strictly complementary goods. The two most used forms of preference are:

- $U = \min(Q_{tr}, Q_{to})$ , for a strict complementarity of the two goods,
- $U = U(Q_{tr}, Q_{to}) - (p_{tr}^* Q_{tr} + p_{to}^* Q_{to})$ , the quasi-linear and quadratic form (Cf. Singh et Vivies [32] and Amir and al. ii [33]), for modular complementarities.  $U(Q_{tr}, Q_{to}) = \theta_{tr} Q_{tr} + \theta_{to} Q_{to} - (\mu_{tr} Q_{tr}^2 + 2\gamma Q_{tr} Q_{to} + \mu_{to} Q_{to}^2)/2$ , with  $\gamma < 0$  measuring the intensity of the goods complementarity.

Formally, the parameters of the utility function are adjusted such that  $Q_{tr} = Q_{to}$  in order to secure the equality of flows transported and received. This adjustment indicates the type of visitors likely to be interested in the destination and the transport to access it, and it expresses a strong destination-transport-visitor relationship. It confirms the imperfect nature of the tourism market, where every destination is not infinitely substitutable, each one corresponding to a type of customer. Two possible forms of tourist flow ( $F$ ) come out of the aforementioned utility functions and the tourist visitor's optimizing program:

- $F = R/(p_{tr}^* + p_{to}^*)^{-1}$ , with the strict complementary good function:  $\min(Q_{tr}, Q_{to})$ . Then  $R$  is the nominal tourist's budget for the two sector-firms.  $F$  tends to infinity when prices tend towards zero (and inversely);

- $F = \alpha - \beta(p_{tr}^* + p_{to}^*)$  is a linear tourism flow function once the parameters of the quasi linear, quadratic utility function are adjusted such that  $Q_{tr} = Q_{to}$ . Singh and Vives [32] provide the analytical form of the demand functions of each good, from which it is easy to adjust the parameters to secure the equality  $Q_{tr} = Q_{to}$ .  $\alpha$  and  $\beta$  are combinations of the utility function parameters ( $\mu, \theta, \gamma, \gamma$ ).  $\beta$  is necessarily positive [ $\beta > 0$ , because  $p_i^*$  are negatively related to capacities ( $T_i$ )] and  $\alpha$  is the maximum attendance (or potential market expressed in number of visitors) when prices tend towards zero.

Considering the two possible flow functions, attendance is a function of the unit price of the transport-tourism activities bundle:  $P = p_{tr}^* + p_{to}^*$  i.e., the amounts spent by visitors on transportation and tourism; hence  $F = F(p_{tr}^* + p_{to}^*)$ . Combined with Equation (1), the general form of the tourist flow is:

$$F = F[P] = F[Q_{tr}^{-1}(T_{tr}) + Q_{to}^{-1}(T_{to})] \quad (3)$$

The increase in capacity implies an increase in tourist flow (under the constraint  $F \leq \min(T_{tr}, T_{to}) \Rightarrow F \leq \frac{T_{to} + T_{tr} - |T_{to} - T_{tr}|}{2}$ ):  $\frac{\partial F}{\partial T_i} = \frac{\partial F}{\partial P} \frac{\partial Q_i^{-1}}{\partial T_i} > 0$  as  $\frac{\partial F}{\partial P} < 0$  and  $\frac{\partial Q_i^{-1}}{\partial T_i} < 0$ . From Equation (3) the tourist flow dynamic equations is:

$$\frac{dF}{dt} = \dot{F} = \frac{\partial F}{\partial P} \left[ \frac{\partial Q_{tr}^{-1}}{\partial T_{tr}} \dot{T}_{tr} + \frac{\partial Q_{to}^{-1}}{\partial T_{to}} \dot{T}_{to} \right] \quad (4)$$

Equations (3) and (4) express the sub-dynamic 1 and 2 of the systemic dynamic approach of destination tourism development as depicted by **Figure 1**. They provide an equation form for tourism development resulting from “by transport” and “by activities/product” dynamics. Equation (3) is static instantaneous, short term flow of tourists and Equation (4) its law of motion for the long run.

Replacing Equation (3) into the sector-firms profit functions and occupancy rate, gives  $\dot{K}_i = h_i(\pi_i, \psi_i) = h_i(T_{tr}, T_{to})$ , hence the final version of Equation system (2):

$$\dot{T}_i = \left( \frac{\delta G_i^{-1}}{\delta T_i} \right)^{-1} h_i(T_{tr}, T_{to}) \quad (5)$$

This dynamic equation system formalizes sub-dynamic n°3, the supply dynamic of the systemic dynamic approach to tourism development. It links the time evolution of one sector's capacity to that of the other. Although each equation includes the other sector-firm capacity, the system cannot be considered a reaction function system. Firstly, because the equations are not the result of an optimization process, they provide the best answers according to other agents' conjectured behavior. Secondly, from the standpoint of sector firms, the integration of each other's capacity is involuntary, induced by the presence of the “involuntary” coordinator visitor. The system does not depict strategic relations among the sector firms but rather systemic relations between the three agents, created by tourism.

### 2.1.2. Interactions

The respective behavior of the three agents leads to interactions in the form of information exchanges, that generate a long-term dynamic. Prices ( $p_i$ ) and capacities ( $T_i$ ) are the basis of interactions. The mechanics of agents' interactions can be read as follows:



In the short run, during the reference period,

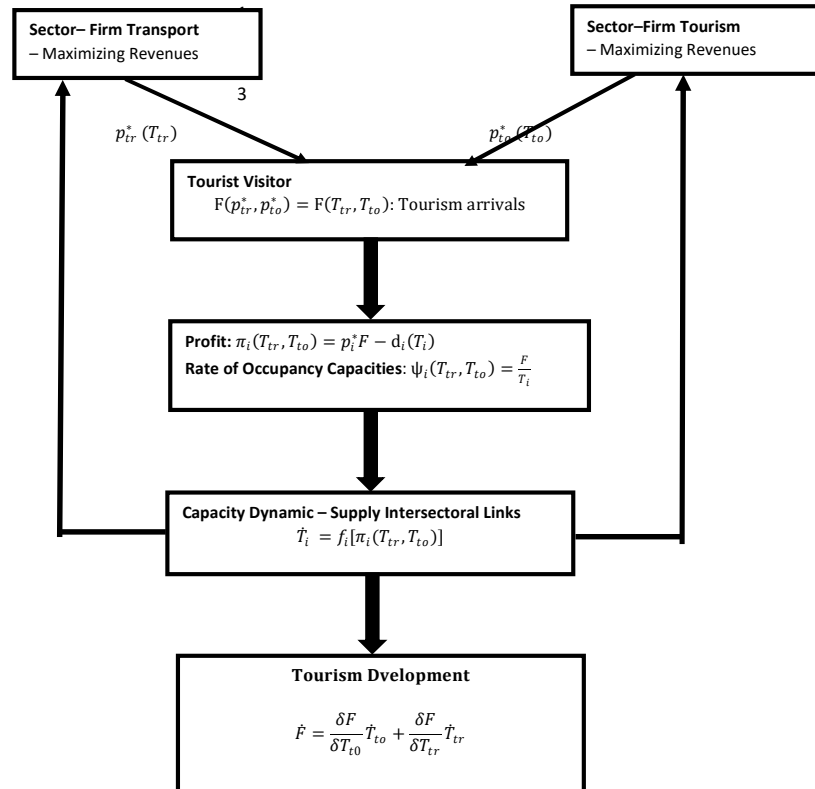
- 1) each sector-firm sets its price ( $p_i^*$ ) to maximize its revenue, inducing prices dependent on sectoral capacities ( $p_i^*(T_i)$ ; Equation (1))
- 2) Producer prices ( $p_i^*$ ) establish the price of the transport-tourism bundle ( $P = p_{tr}^* + p_{to}^*$ ) and consequently tourism flow ( $F$ ), according to Equation (3) [ $F = F(p_i^*) = F[p_i^*(T_i)] = F(T_{tr}, T_{to})$ ]. Sectors' capacities are not necessarily entirely used, and the tourist visitors appear as the period coordinating agent of the two sector-firms [this coordinating role is facilitated by information technology (via platforms or directly with each sector), which allows tourist visitors to bundle themselves the Transport-Tourism products. It reduces the influence of intermediaries (TO, agencies...)].

In the reference period, all the descriptive variables of destination tourism development depend on capacities ( $T_i$ ).

The law of motion of capacities, the capacity dynamic, triggers tourism development:

- 3) According to Equation (4):  $\dot{F} = \frac{\partial F}{\partial P} \left[ \frac{\partial Q_{tr}^{-1}}{\partial T_{tr}} \dot{T}_{tr} + \frac{\partial Q_{to}^{-1}}{\partial T_{to}} \dot{T}_{to} \right]$
- 4) Under the control of the model's inter-sector interactions, according to Equations (7) and (8).

**Figure 2** depicts the interactions of the model.



**Figure 2.** Systemic dynamic capacity-based model of destination tourism development.

Based upon 4 behavioral assumptions of tourism visitors and of supply sector-firms, a systemic dynamic approach of destination tourism development can be

formalized by the following three dynamic equations system:

$$\dot{F} = \frac{\partial F}{\partial P} \left[ \frac{\partial Q_{tr}^{-1}}{\partial T_{tr}} \dot{T}_{tr} + \frac{\partial Q_{to}^{-1}}{\partial T_{to}} \dot{T}_{to} \right] \quad (6)$$

$$\dot{T}_{tr} = \left( \frac{\delta G_{tr}^{-1}}{\delta T_{tr}} \right)^{-1} h_{tr}(T_{tr}, T_{to}) \quad (7)$$

$$\dot{T}_{to} = \left( \frac{\delta G_{to}^{-1}}{\delta T_{to}} \right)^{-1} h_{to}(T_{tr}, T_{to}) \quad (8)$$

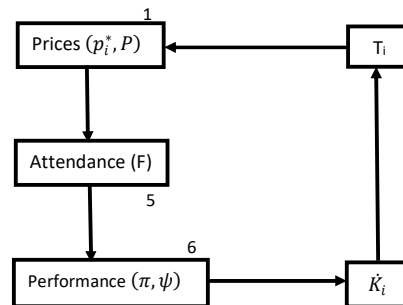
The equations system can also be expressed in a recursive form.

## 2.2. A few learnings from the model

The model provides a general understanding of destination tourism development. It enables the identification of structural variables of its mechanic. It also helps to diagnose situations that may halt destination tourism development.

By modelling destination tourism development through a dynamic equations system, the capacity-based model allows to conceptualize destination tourism development as a microfounded supply-driven systemic dynamic process. Microfounded because, destination tourism development is the result of agents' behavior, involved in the process. Visitors are considered rational and optimizing. Similarly, sector firms are regarded as rational and optimizing for setting their prices, and as adopting conventional investment behavior. Destination tourism development is also supply-driven because the main impetus of tourism attendance comes from sector-firm production capacity ( $T_i$ ) and their time law of motions ( $T_{tr}, T_{to}$ ), as outlined by Equation (4). It is also dynamic, for it is formalized as a dynamic equations system, able to generate a wide spectrum of tourism attendance time paths. Finally, it is systemic because interlinked capacities, its main driving force, generates consequences (tourism time attendance) that do not necessarily spring up out of perfectly informed or projected agents' decisions. In summary, the short-term micro or meso level (transport and tourism activities being considered as sectors) generates a macrodynamic providing the time evolution of the all the variables featuring the tourism system: prices of capacity units [ $p_i^*(T_i)$ ], sectoral capacities ( $T_i$ ), occupation coefficients [ $\psi_i = \frac{F}{T_i}$ ], sectoral profits ( $\pi_i$ ), and global receipts at the destination level.

Through the prism of the model, destination tourism development can be structurally conceptualized as a system relating four variables or indicators: price, capacity, tourism attendance, and performance, as depicted in **Figure 3**:



**Figure 3.** Tourism dynamic system variables.

Visitors' sensibility to price and sector-firms' pricing behavior to capacity changes  $[\frac{\partial Q_i^{-1}}{\partial T_i}, \frac{\partial F}{\partial P}]$ , as well as capacity production technologies  $[\frac{\delta G_i^{-1}}{\delta T_i}]$  are structuring parameters of destination tourism development. Together with the performance functional forms, they decide the speed, modulate, or shape destination tourism development trajectory, as they are multiplicative factors of the equations system. In a technologically stable situation, the model identifies pricing reactions to capacity changes by sector-firms as the primary tools for monitoring destination tourism development.

The model offers a large spectrum of possible destination tourism development paths: linear, non-linear, logistic, stable or not, converging or chaotic, according to the structuring parameters and functional forms of performance. Thus, it could be used for simulation, forecasts, and to understand agents' behavior through estimating structuring parameters. The capacity-based model allows to identify two situations that can stop destination tourism development:

- When changes in the optimal prices of sector-firms offset each other: each change in the optimal price of a sector-firm is the exact opposite of the change in the optimal price of the other  $\frac{dp_{tr}^*}{dp_{to}^*} = -1$  [From the bundle transport-tourism activities price,  $P = p_{tr}^*(T_{tr}) + p_{to}^*(T_{to})$ , it comes  $\dot{P} = \frac{dp_{tr}^*}{dT_{tr}} \dot{T}_{tr} + \frac{dp_{to}^*}{dT_{to}} \dot{T}_{to}$ , with:  $\dot{P} = 0 \Rightarrow \frac{dp_{tr}^*}{dp_{to}^*} = -\frac{dT_{tr}/dt}{dT_{to}/dt} \frac{\dot{T}_{to}}{\dot{T}_{tr}} \Leftrightarrow \frac{dp_{tr}^*}{dp_{to}^*} = -1 \Leftrightarrow \dot{p}_{tr}^* = \dot{p}_{to}^*$ . A constant  $P$  means a constant  $F$ , and  $\dot{F} = 0$  and  $\frac{\partial F}{\partial P}[0] = 0$ , according to Equation (4)]. Dynamic stability aside, a constant bundle of transport-tourism activities' price describes an asymmetric situation where one sector exhausts all the destination's market power by increasing its price, leaving the other sector to decrease its price to maintain tourist attendance. This situation also means an opposite time variation of capacities: one sector-firm reduces its capacity while the other increases its  $[\frac{dp_{tr}^*}{dT_{tr}} = -\frac{dp_{to}^*}{dT_{to}} \frac{dT_{to}}{dT_{tr}} \Rightarrow \frac{dT_{to}}{dT_{tr}} < 0]$ ;
- Destination tourism development also turn off when sector-firms' capacities dynamic simultaneously cease (Equations (6) and (7),  $\dot{T}_{tr} = \dot{T}_{to} = 0$ ) which imply constant capacities ( $T_i$ ) and consequently constant attendance and prices ( $F$ ,  $p_{to}^*$ ,  $p_{tr}^*$  and  $P$ ). It implies a specific relation (or a given capacity ratio) between sector-firms capacities (from  $(\frac{\delta G_{tr}^{-1}}{\delta T_{tr}})^{-1} h_{tr}(T_{tr}, T_{to}) = 0$ , and  $(\frac{\delta G_{to}^{-1}}{\delta T_{to}})^{-1} h_{to}(T_{tr}, T_{to}) = 0$ . The simultaneous nullity of capacity dynamics ( $T_i = 0$ ) implies the nullity of the performance function  $[h_i(\pi_i, \psi_i) = 0$  with  $h_i(0) = 0]$ . This can be the result of 0 profit in each sector-firm: a situation revealing the exhaustion of profit, that disappears with the continuous increase in capacity ( $T_i$ ), as signaled by monopolistic competition theory.

The common feature of those situations of tourism non-development is linked to the market power created by imperfection competition on the tourism market: its exhaustion or its asymmetric distribution may end destination tourism development.

The following section exposes the ability of the capacity-based model to generate

a TALC-compliant logistics destination tourism development path and helps to better understand it from a microfoundation perspective.

### 3. The talc as a capacity-based model of destination tourism development

There are some rationales to model the TALC with the capacity-based dynamic systemic model previously presented. Firstly, modeling the TALC through a unique logistic differential equation makes it a black box-type macro model. The capacity-based model allows TALC to be viewed as a supply-side process rather than a macro-demand process, as is thought by tourism planners and some academic work. Moreover, it proposes a supply-side microfounded understanding. Secondly, the deceleration and acceleration forces listed by Butler [34] have rarely been modeled. Very few attempts to model the determinants of the TALC using dynamic systems have been made [35–40]. Thirdly, despite the results of Kato [41] (which point out the importance of technical progress in the transport sector for the logistical TALC profile of tourist flows in the case of Hawaii), and to our best knowledge, no paper has specifically modeled the influence of transport in the TALC framework.

The logistic path is a possible trajectory of the capacity-based tourism development model. By specifying tourists and sector firms' behaviors and technology, the model allows replication of TALC's logistical trajectory for tourist attendance. As such, the model enables us to microfound the TALC; it provides a theoretical framework for understanding the TALC from microeconomic behaviors.

According to the capacity-based model, tourism attendance follows a logistic curve, if Equation (4)  $\dot{F}$  is:

- A degree 2 quadratic function with two variables (polynomial equation of degree 2, which generic writing is:  $AT_{to}^2 + BT_{tr}^2 + CT_{to} + DT_{tr} + ET_{to}T_{tr} + C$ ;
- Conform to an elliptical parabola (inverted U-shaped with a single inflection point) ideally with an initial point equal to the nullity (at time  $t_0$ ,  $T_{tr}=T_{to}=0$  since without capacity there can be no tourist flow), requiring:
  - $4AB - E^2 > 0$ , with  $A$  and  $B < 0$  (or in the case where  $4AB = E^2$ , if  $DE - 2CB = 2AD - CE = 0$ ).

These conditions are met when technology and agents' behaviors are linear. The combination of:

- 1) A constant return to scale linear production technology, under assumption 3, implying  $K_i = v_i T_i$  with a constant  $\left(\frac{\delta G_{tr}^{-1}}{\delta T_{tr}}\right)^{-1}$  inducing a linear total cost function (for example, an AK-type technology [Romer (1987), Rebelo (1991)] gives  $T_i = A_i K_i$  and induces  $\left(\frac{\delta G_{tr}^{-1}}{\delta T_{tr}}\right)^{-1} = A$ , with a Total Cost Function:  $CT = d_i K_i = d_i \frac{T_i}{A_i}$ );
- 2) With a linear sector-firms' pricing behavior:  $Q_i^{-1}(T_i) = p_i^* = a_i - b_i T_i$ , generating a constant  $\frac{\partial p_i^*}{\partial T_i}$ ;
- 3) With tourist linear demands functions entailing a linear tourist flow function:  $F =$

$\alpha - \beta(p_{tr}^* + p_{to}^*)$  producing a stable  $\frac{\partial F}{\partial T_i} = \beta$ ;

- 4) With a linear investment behavior linearly relating  $\dot{K}$  with performance indicators ( $\pi$  and/or  $\psi$ ) and more generally with sector-firms revenues:

Necessarily formalize  $\dot{F}$  as quadratic function. Conditions 1, 2 and 3 produce quadratic revenue and profit functions for the sector-firms, as exposed below, with  $F = \beta(b_{to}T_{to} + b_{tr}T_{tr})$ , the meaning and consequences of which is explained in Appendix B:

$$\begin{aligned}\pi_{to} &= \underbrace{(a_{to} - b_{to}T_{to})[\beta(b_{to}T_{to} + b_{tr}T_{tr})]}_{\text{Revenue}} - \underbrace{T_{to}/v_{to}}_{\text{Cost}} \\ &= \underbrace{a_{to}\beta b_{to}T_{to} + a_{to}\beta b_{tr}T_{tr} - \beta b_{to}^2T_{to}^2 - \beta b_{to}b_{tr}T_{to}T_{tr}}_{\text{Revenue}} - \underbrace{T_{to}/v_{to}}_{\text{Cost}} \\ \pi_{tr} &= \underbrace{(a_{tr} - b_{tr}T_{tr})[\beta(b_{to}T_{to} + b_{tr}T_{tr})]}_{\text{Revenue}} - \underbrace{T_{tr}/v_{tr}}_{\text{Cost}} \\ &= \underbrace{a_{tr}\beta b_{to}T_{to} + a_{tr}\beta b_{tr}T_{tr} - \beta b_{tr}^2T_{tr}^2 - \beta b_{to}b_{tr}T_{to}T_{tr}}_{\text{Revenue}} - \underbrace{T_{tr}/v_{tr}}_{\text{Cost}}\end{aligned}$$

Condition 4 expresses the linear link between investment and profit or revenue:  $\dot{K}_i = h_i(\pi_i, \psi_i) = m_i \times \pi_i$  or  $m_i \times \text{Revenue}_i$  [Revenue = Potential Revenue  $p_i^*(T_i) \times T_i \times \psi \left( = \frac{F}{T_i} \right)$ ], with  $m$  measuring the sensibility of investment to profit (or revenue) or the percentage of profit (or revenue) dedicated to fund investment.

Considering the linearity conditions, attendance time motion becomes a quadratic function like the one below (with revenue only for simplicity):

$$\dot{F} = \beta[a_{to}(v_{to})^{-1}m_{to}(a_{to}\beta b_{to}T_{to} + a_{to}\beta b_{tr}T_{tr} - \beta b_{to}^2T_{to}^2 - \beta b_{to}b_{tr}T_{to}T_{tr}) + a_{tr}(v_{tr})^{-1}m_{tr}(a_{tr}\beta b_{to}T_{to} + a_{tr}\beta b_{tr}T_{tr} - \beta b_{tr}^2T_{tr}^2 - \beta b_{to}b_{tr}T_{to}T_{tr})].$$

$\dot{F}$  is quadratic because  $[a_{tr}(v_{to})^{-1}m_{tr}a_{tr}(v_{tr})^{-1}m_{tr}]^2[4\beta^2b_{tr}^2b_{to}^2 - \beta^2b_{tr}^2b_{to}^2] > 0$ .

The capacity dynamic model of TALC implies a continuous increase in the aggregate capacity of transport and tourism activities and induces a steady decline in their individual prices and, consequently, in the bundle price. This regular decline in price, driven by the increase in capacity, is the impetus for the growth in tourist numbers until the stagnation period with equilibrium prices and capacities. All things being equal, the model allows us to identify the linear behaviors of sector firms and tourists necessary to generate a TALC tourism development process. Linear behaviors entail changing elasticities over time. Thus, attendance is more elastic to bundle prices early in the development process (before half the potential market). Similarly, sector-firms prices are inelastic to new capacities ( $|ep_{i_{T_i}}| < 1$ ) and capacity-elasticity of price decreases over the course of tourism development, making “rejuvenation” a possible issue for competition regulation. In summary, the model highlights the evolution of capacities as the accelerating force in TALC (increasing arrivals), itself generating its own decelerating force: decreasing price elasticities (a declining flow due to a lesser impact of capacities on prices).

#### 4. Concluding remarks

The international definition of tourism prompts the development of a dynamic systemic approach to tourism development underpinned by three dynamics: a

transport-demand dynamic, a product/activity-demand process, and a supply-side transport-activity dynamic. Inspired by the dynamic systemic approach and with a minimalist set of hypotheses, it is possible to derive a capacity-based model that enables us to consider destination tourism development as a microfounded supply-side systemic dynamic process. Destination tourism development can be structurally conceptualized as a system relating four variables: price, attendance, and performance, all dependent on the fourth: sector-firm capacity of transport and tourism activities. The dynamic of the system, and consequently of destination attendance in time, is monitored by parameters expressing visitors' sensibility to bundle price, sector-firm pricing behavior to capacity changes, and production technology of capacities. The model offers a large spectrum of possible destination tourism development paths. As such, it can be used for simulation, forecasts, understanding agents' behavior, and providing tools for tourism planners. Also, it highlights exhaustion or asymmetric distribution of market power as two situations able to halt destination tourism development. The capacity-based model of destination tourism development identifies the double impact of capacity dynamics on TALC forces: accelerating by increasing arrivals and, at the same time, decelerating by declining price elasticities.

**Conflict of interest:** The author declares no conflict of interest.

## References

1. ONU, OMT, OCDE, Eurostat. Tourism satellite account: recommendations for the 2008 conceptual framework (French). Luxembourg, Madrid, New York, Paris. 2008.
2. Breakey NM. Tourism destination development-beyond butler. Available online: <https://core.ac.uk/download/pdf/15055094.pdf>.
3. Butler RW. The concept of a tourist area cycle of evolution: implications for management of resources. Canadian Geographies/GÉographies Canadiennes. 1980, 24(1): 5-12. doi: 10.1111/j.1541-0064.1980.tb00970.x
4. Lim C. A survey of tourism demand modelling practice: issues and implications. In: Dwyer L, Forsyth P (editors). International Handbook of tourism economics. Cheltenham UK: Edward Elgar. 2006. pp. 45-72.
5. Stabler MJ, Papatheodorou A, Sinclair T. The Economics of Tourism. New York: Routledge. 2010.
6. Li G, Song H, Witt SF. Recent Developments in Econometric Modeling and Forecasting. Journal of Travel Research. 2005, 44(1): 82-99. doi: 10.1177/0047287505276594
7. Sinclair T, Stabler M. The Economics of Tourism. 3rd ed. London: Routledge. 2002.
8. Deaton A, Muellbauer J. An Almost Ideal Demand System. The American Economic Review. 1980; 3: 312-326.
9. Lancaster KJ. Consumer Demand: A New Approach, Columbia: New York: University Press. 1971.
10. Anas A. Taste heterogeneity and urban spatial structure: The logit model and monocentric theory reconciled. Journal of Urban Economics. 1990, 28(3): 318-335. doi: 10.1016/0094-1190(90)90031-h
11. Morley CL. Experimental destination choice analysis. Annals of Tourism Research. 1994; 21: 780-791.
12. Alegre J, Pou L. The length of stay in the demand for tourism. Tourism Management. 2006, 27(6): 1343-1355. doi: 10.1016/j.tourman.2005.06.012
13. Son Song H, Dwyer L, Li G, et al. Tourism economics research: A review and assessment. Annals of Tourism Research. 2012, 39(3): 1653-1682. doi: 10.1016/j.annals.2012.05.023
14. Son Song H, Witt SF, Li G. The Advanced Econometrics of Tourism Demand. London: Routledge. 2009.
15. Divisekera S. Empirical Estimation of Tourism Demands Models: A review. In: Tisdell CA (editor). Handbook of Tourism Economics. Singapore: World Scientific Publishing CO. Pte. Ltd. 2003. pp. 67-86.
16. McIntosh RN, Goeldner CR, Ritchie JR. Tourism, Principles, Practices, Philosophies. 7th ed. New York: John Wiley & Sons. 1995
17. Prideaux B. The role of the transport system in destination development. Tourism Management. 2000, 21(1): 53-63. doi:

- 10.1016/s0261-5177(99)00079-5
18. Lumsdon L, Page S. *Tourism and transport: issues and agenda for the new millennium*. New York, USA: Routledge. 2004.
  19. Gay JC. *Transport and tourism of the world (French)*. Collection EDYTEM Cahiers de géographie. 2006, 4(1): 11-22. doi: 10.3406/edyte.2006.960
  20. Page S. *Transport and Tourism: Global perspectives*. 3rd, ed. Harlow, England: Pearson Education Limited. 2009.
  21. Lohmann G, Duval DT. *Critical Aspects of the Tourism-Transport Relationship*. Contemporary Tourism Reviews. Oxford, UK: Goodfellow Publishers. 2011.
  22. UNWTO. *Global Report on Aviation: Responding to the needs of new tourism markets and destinations*. Madrid, Spain: World Tourism Organization (UNWTO). 2012
  23. Durand D. *Systemics (French)*. 13 ed. Presse Universitaire de France; 2017.
  24. Candela G, Figini P, Scorcu AE. *The Economics of Local Tourism Systems*. In: Brau RA., Lanza S (editors). *Usai Tourism and Sustainable Economic Development: Macroeconomic Models and Empirical Methods*. Cheltenham, Edward Elgar. 2008.
  25. Candela G, Figini P. Destination unknown. Is there any economics beyond tourism areas? *Review of Economic Analysis*. 2010; 2: 256-271.
  26. Andergassen R, Candela G, Figini P. An economic model for tourism destinations: Product sophistication and price coordination. *Tourism Management*. 2013, 37: 86-98. doi: 10.1016/j.tourman.2012.10.013
  27. Alexiou C. A Keynesian-Kaleckian model of investment determination: a panel data investigation. *Journal of Post Keynesian Economics*. 2010, 32(3): 427-444. doi: 10.2753/pke0160-3477320307
  28. Stevens GVG. Internal Funds and the Investment Function. *Southern Economic Journal*. 1994, 60(3): 551. doi: 10.2307/1060566
  29. Fazzari SM, Mott TL. The Investment Theories of Kalecki and Keynes: An Empirical Study of Firm Data, 1970–1982. *Journal of Post Keynesian Economics*. 1986, 9(2): 171-187. doi: 10.1080/01603477.1986.11489611
  30. Samuel C. Internal finance and investment: another look. *The World Bank Policy Research Working Paper Series* 1663. 1996.
  31. Bull AO. Industrial Economics and Pricing Issues within Tourism Enterprises and Markets. In: Dwyer L, Forsyth P (editors). *International Handbook of Tourism Economics*. Cheltenham UK: Edward Elgar. 2006. pp. 138-154.
  32. Singh N, Vives X. Price and Quantity Competition in a Differentiated Duopoly. *The RAND Journal of Economics*. 1984, 15(4): 546. doi: 10.2307/2555525
  33. Amir R, Erickson P, Jin J. On the microeconomic foundations of linear demand for differentiated products. *Journal of Economic Theory*. 2017, 169: 641-665. doi: 10.1016/j.jet.2017.03.005
  34. Butler RW. *Tourism area life cycle Vol.1, Vol. 2*, Channel View Publications, Clevedon, United Kingdom—New York, USA—North York, Ontario, Canada. 2006.
  35. Lundtorp S, Wanhall S. The resort lifecycle theory. *Annals of Tourism Research*. 2001, 28(4): 947-964. doi: 10.1016/s0160-7383(00)00080-3
  36. Lozano J, Gómez CM, Rey-Maqueira J. The TALC Hypothesis and Economic Growth Theory. *Tourism Economics*. 2008, 14(4): 727-749. doi: 10.5367/000000008786440166
  37. Johnston RJ, Tyrrell TJ. A Dynamic Model of Sustainable Tourism. *Journal of Travel Research*. 2005, 44(2): 124-134. doi: 10.1177/0047287505278987
  38. Casagrandi R, Rinaldi S. A Theoretical Approach to Tourism Sustainability. *Conservation Ecology*. 2002, 6(1). doi: 10.5751/es-00384-060113
  39. Faria JR. Demographic and Technological Growth in the Tourism Market. *Tourism Economics*. 2008, 14(1): 115-121. doi: 10.5367/000000008783554776
  40. Li J, Chen Z. A New Interpretation of the Model of Tourism Life Cycle: Based on Production Investment and Demand Analyses. *Tourism Tribune/LvyouXuekan*. 2014; 29(3): 58-72.
  41. Kato A, Mack J. Technical Progress in Transport and the Tourism Area Life Cycle. In: Tisdell CA (editor). *Handbook of Tourism Economics*. Singapore: World Scientific Publishing CO. Pte. Ltd. 2003. pp. 67-86.

## Appendix A

Sector-firms' pricing behavior:

From the sector-firms optimization program  $\text{Max}_{p_i} Q_i(p_i)$  Under constraint,  $T_i \geq Q_i(p_i)$  follows the optimality conditions:

- $Q_i(p_i) - p_i \left| \frac{\delta Q_i}{\delta p_i} \right| + \lambda \left| \frac{\delta Q_i}{\delta p_i} \right| = 0,$
- $T_i \geq Q_i(p_i),$
- $\lambda \geq 0, \lambda [T_i - Q_i(p_i)] = 0.$

This leads to retaining the constraint saturation  $[T_i = Q_i(p_i)]$ , since a Lagrangian equal to nullity imposes a relative inelasticity of demand that undermines the generality of the model.  $\lambda = 0$  satisfies the second condition and  $\Rightarrow Q_i(p_i) = p_i \left| \frac{\delta Q_i}{\delta p_i} \right| = 0$ , hence  $T_i \geq p_i \left| \frac{\delta Q_i}{\delta p_i} \right| \Rightarrow \frac{T_i}{Q_i(p_i)} \geq \frac{p_i}{Q_i(p_i)} \left| \frac{\delta Q_i}{\delta p_i} \right| = e_i \Leftrightarrow e_i \leq 1$ . Therefore  $\lambda \geq 0$ , the optimizing behavior of producers sets out pricing as follows:

$$p_i^* = Q_i^{-1}(T_i) \quad (1)$$

where  $Q_i^{-1}$  is the inverse function of  $Q_i$ , with  $\frac{\delta Q_i^{-1}}{\delta T_i} < 0$ ; derivatives of inverse functions having the same slope as their initial functions.



## Appendix B

Attendance function without intercept and elasticities:

For  $F$  be written without intercept and starts from 0, it is necessary that:  $\alpha - \beta(a_{to} - b_{to}T_{to} + a_{tr} - b_{tr}T_{tr}) = \beta(b_{to}T_{to} + b_{tr}T_{tr})$  and thus that  $\alpha = \beta(a_{to} + a_{tr})$ . This condition implicitly expresses that the potential market of the tourist destination ( $\alpha$ ) is not only dependent on demand characteristics but also on maximum capacities. Indeed, as  $p_i > 0$ , the maximum capacity is reached when  $p_i = 0$ , i.e.,  $T_i \text{ Maximum} = a_i/b_i \Leftrightarrow a_i = b_i \times \max T_i$ , hence  $\alpha = \beta(b_{to}\max T_{to} + b_{tr}\max T_{tr})$ . Thus, the potential market of the destination ( $\alpha$ ) depends partly on the characteristics of the demand ( $\beta$ ) and the maximum capacities ( $\max T_{to}$ ;  $\max T_{tr}$ ).

$F = \beta(b_{to}T_{to} + b_{tr}T_{tr})$  says that tourist flow is 0 when simultaneously there are no transport and tourism capacities; the occurrence of a single capacity,  $T_{tr} \neq 0$  or  $T_{to} \neq 0$ , makes it possible to initiate the capacity dynamics.

Considering the flow function  $[\beta(b_{to}T_{to} + b_{tr}T_{tr})]$  the inferiority constraint of attendance relative to the capacities ( $\psi_i < 1$ ) is satisfied if  $\beta b_{to} \leq 1$  and  $\beta b_{tr} \leq 1$ , as exposed below:

$$F \leq \min(T_{tr}, T_{to}) \Rightarrow \beta(b_{to}T_{to} + b_{tr}T_{tr}) \leq T_{tr} \text{ or } \beta(b_{to}T_{to} + b_{tr}T_{tr}) \leq T_{to} \Rightarrow (\beta b_{to} - 1)T_{to} \leq -\beta b_{tr}T_{tr} \text{ or } (\beta b_{tr} - 1)T_{tr} \leq -\beta b_{to}T_{to}.$$

Hence, whenever  $T_{to} \leq T_{tr}$ , the inferiority constraint of attendance condition is satisfied if  $\beta b_{to} \leq 1$  and  $\beta b_{tr} \leq 1$ . These are necessary and sufficient conditions if the initial zero capacities ( $T_{tr} = T_{to} = 0$ ) converge to positive maximum equilibrium capacities ( $T_{tr}^*$ ;  $T_{to}^*$ ).

Then, if the constraint  $F < \min(T_{tr}, T_{to})$  holds at each instant of time, it comes  $\dot{F} \leq \dot{T}_i$  and consequently elasticity-capacity of prices and flows are necessarily lower than unity since:

- $ep_{i T_i} = \frac{\delta p_i T_i}{\delta T_i p_i} = -b_i \frac{T_i}{p_i} = \frac{-b_i T_i}{a_i - b_i T_i} \Leftrightarrow |ep_{i T_i}| < 1$ ;
- $eF_P = \frac{\delta F}{\delta P} \frac{P}{F} = \beta \frac{a_{to} + a_{tr} - (b_{to}T_{to} + b_{tr}T_{tr})}{\beta(b_{to}T_{to} + b_{tr}T_{tr})} \Rightarrow eF_P > 1 \Rightarrow \frac{\alpha}{2} > F$ , as  $F$  is supposed to start at 0. This means that the capacity elasticity of attendance is greater than unity up to half the potential market.