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Utilization of complex networks for analyzing PM_{2.5} air pollution in various regions

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Copyright © 2020 by author(s). Pollution Study is published by Asia Pacific Academy of Science Pte. Ltd. This work is licensed under the Creative Commons Attribution (CC BY) license. https://creativecommons.org/licenses/ **Abstract:** The complex network approach was employed to investigate PM_{2.5} levels in air pollution. Relevant PM_{2.5} data were analyzed for correlation, leading to the establishment of a complex network across various regions in China. By examining factors such as degree, community structure, and motifs, the findings indicated that this method effectively identifies the major polluted cities in China. Furthermore, cities experiencing clustered air pollution should be addressed collectively, reflecting the actual conditions. Given the dynamic nature of air movement, this research offers valuable insights for analyzing the aggregation of polluted cities.

Keywords: complex network; degree; community structure; motif

In recent years, with the rapid development of China's economy, the acceleration of industrialization and population growth, energy consumption is too fast, resulting in air pollution. China's air quality shows a downward trend, and smog weather occurs frequently. The main component of smog is atmospheric particulate matter, namely PM_{2.5}, whose concentration greatly exceeds the range specified by the state. The main harm of air pollution comes from PM_{2.5}, which causes serious damage to people's respiratory system, nervous system and skin tissue [1]. Therefore, pm_{2.5} concentration index is an important detection index of ambient air quality. How to effectively control air pollution is an urgent problem to be solved.

It is worth noting that each heavy pollution is not an individual city, but covers multiple cities. The regional air pollution is not only related to the local pollution of the city, but also related to the air circulation habits of other cities near the region. It is urgent to control air pollution. It is necessary to carry out research from various aspects [2,3] such as blocks to alleviate and effectively reduce the national $PM_{2.5}$ index.

PM_{2.5} is a very fine particle, which cannot be shielded by the respiratory tract. These fine particles will enter the respiratory tract and lungs when people are unprepared. Induce rhinitis, pharyngitis, bronchitis, asthma and other respiratory diseases. In this environment for a long time, particles entering the lungs may also enter the blood through the alveoli, inducing a series of cardiovascular diseases such as lung cancer, myocardial ischemia, hypertension, etc. The research on the analysis and remediation of PM_{2.5} has always been valued by people [4]. In foreign countries, the research on PM_{2.5} was carried out earlier, and the research on the source analysis and characteristic spectrum of PM_{2.5} was carried out more [5]. In 2014, Elangasinghe et al. [6] used the method of artificial neural network for analysis. The research in China started relatively late. At first, the statistical data on PM_{2.5} were few and

incomplete. In recent years, the data has been gradually accumulated and unified, which is of great significance for effective and reasonable research on the very limited air data [7]. Yang et al. [8] analyzed the chemical composition and source of PM_{2.5}. Sun et al. [9] analyzed the concentration in winter and summer in Beijing and found that the concentration in winter was higher in Beijing. Yu et al. [10] and Song et al. [11] proposed that there is a negative correlation between PM_{2.5} and visibility of air quality in Beijing. Su et al. [12] proposed that Beijing has external transmission channels for air pollutants.

To reduce the PM_{2.5} index, first of all, it is necessary to select the main pollution source cities and reduce several seriously polluted areas. In this way, pollution control can draw inferences from one instance. How to find out the main pollution source urban areas through the PM_{2.5} index of each city? In this paper, the complex network method is used to analyze the air pollution areas. Most complex systems in nature can be described by complex networks. Generally, nodes represent different individuals in a real system, and edges represent some kind of connection between individuals. The research and analysis results of complex networks can deepen the understanding of many complex systems in the real world.

In recent decades, the research of complex networks has developed rapidly. There are many articles on complex networks, and the research objects of complex networks are diverse. Complex networks have gradually become a hot research field in the interdisciplinary fields of finance, sociology, computer science, information science and so on [13–20]. However, there are few studies on air quality using complex networks, especially on network models [21].

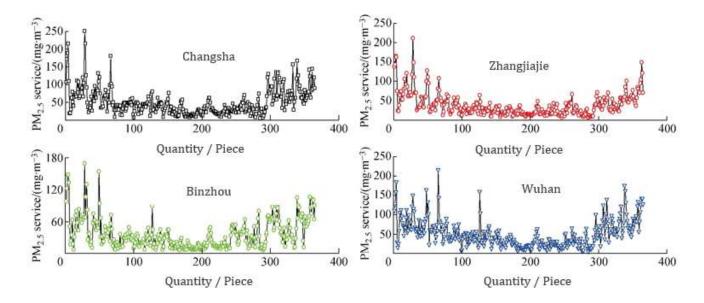
Wang [22] took the air quality data of 8 monitoring points in Beijing as the research object, reconstructed the phase space of the time series of air quality, and built a complex network. Using k-means clustering analysis method, three kinds of clustering results of air quality in Beijing are obtained. Zhang et al. [23] analyzed the role of various factors in the urban environment, explored the physical process of PM_{2.5} regional diffusion, constructed the capacity network model of urban PM_{2.5} diffusion by using the shortest augmented chain algorithm based on the complex network theory, and revealed the diffusion path of PM_{2.5} in Xi'an. Xue et al. [24] took 161 cities as the points of the network, took the correlation of PM_{2.5} mass concentration between cities and the ratio of distance as the weight of each side, built a weighted network, and used Girvan Newman algorithm to divide the network, so as to obtain the regional distribution of China's PM_{2.5} pollution in different seasons and the difference of regional division of different seasons according to the regional degree of pollution in different seasons.

This paper analyzes the 365 day PM_{2.5} data of 68 regions in 2017. These 68 regions belong to the five major regions of China, namely, east China, west China, south China, central China and North China. By calculating the correlation between regions, the complex relationship network is obtained. The degree, community and module in the network are analyzed, and the main pollution source areas are obtained. These results can provide a reference for pollution control in major polluted urban areas.

1. Model establishment

1.1. Data

The PM_{2.5} index selected in this paper takes 68 regions in China as the research object. The 68 regions are divided into five categories, which are from East China, north China, central China, south China and West China. There are 20 regions in East China, including Shanghai, Hefei, Huangshan, Bengbu, Nantong, Nanjing, Suzhou, Yangzhou, Hangzhou, Ningbo, Wenzhou, Jiaxing, Fuzhou, Xiamen, Quanzhou, Jinan, Qingdao, Yantai, Nanchang and Jingdezhen (the serial numbers are 1-20). There are [24] 18 regions in North China, including Beijing, Tianjin, Xilingol, Baoding, Hengshui, Xingtai, Handan, Tangshan, Cangzhou, Zhangjiakou, Chengde, Qinhuangdao, Baotou, Hohhot, Shijiazhuang, Langfang, Taiyuan and Yuncheng (serial numbers 21-38). There are 8 regions in Central China, namely Changsha, Zhangjiajie, Chenzhou, Wuhan, Jingzhou, Zhengzhou, Kaifeng and Luoyang (serial numbers 39-46 respectively). There are 8 regions in South China, namely Guangzhou, Zhongshan, Shenzhen, Guilin, Beihai, Nanning, Haikou and Sanya (serial numbers are 47–54 respectively). There are 14 regions in West China, including Xining, Hainan, Urumqi, Aksu, Turpan, Lanzhou, Tianshui, Lhasa, Chengdu, Mianyang, Chongqing, Yinchuan, Kunming and Lijiang (serial numbers are 55-68). The series length is PM_{2.5} index of all dates from 1 January 2017 to 31 December 2017. The data comes from the weather report. The sequence diagram of central China is shown in Figure 1.



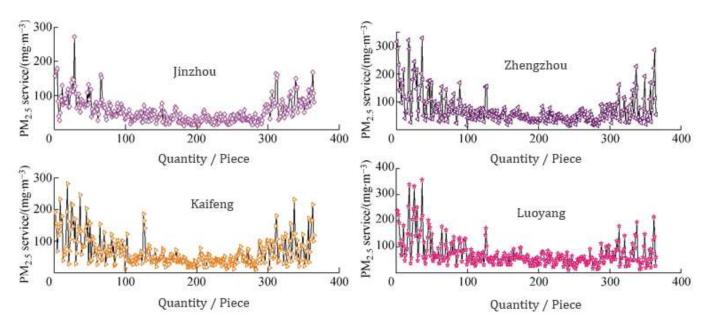


Figure 1. Series of central China.

1.2. PM_{2.5} Data collation and correlation calculation in various regions

Assume that the air pollution data of each region is:

$$Y = \{Y_{ij}, i = 1, 2, \dots, I; j = 1, 2, \dots, J\}$$
 (1)

where, PM_{2.5} Y_{ij} data of the *i*-th region on the *j*-th day, i = 68, j = 365. In order to be comparable, each data is divided by the root mean square of the data in other regions on that day as the de trend processing for the data:

$$X = \{X_{ij}, i = 1, 2, \dots, I; j = 1, 2, \dots, J\}$$

$$X_{ij} = \frac{Y_{ij}}{\sqrt{\sum_{k=1}^{I} \frac{Y_{ij}^{2}}{I}}}$$
(2)

The Pearson correlation coefficient calculated for the data of each region is:

$$\rho_{ij} = \frac{\sum_{j=1}^{J} X_{ij} \cdot X_{ij} - \frac{1}{J} \left(\sum_{j=1}^{J} X_{ij} \cdot \sum_{j=1}^{J} X_{ij} \right)}{\sqrt{\left(\sum_{j=1}^{J} X_{ij}^{2} - \langle \sum_{j=1}^{J} X_{ij}^{2} \rangle \right) \left(\sum_{j=1}^{J} X_{ij}^{2} - \langle \sum_{j=1}^{J} X_{ij}^{2} \rangle \right)}}$$
(3)

where, $\rho_{ij}(\Delta t) \in [-1,1]$ (from complete negative correlation -1 to complete positive correlation 1). If $\rho_{ij}(\Delta t) = 0$ air data is not relevant. The corresponding correlation coefficient matrix is symmetric.

Define distance matrix:

$$d_{ij} = \sqrt{1 - \left| \rho_{ij} \right|} \tag{4}$$

where, d_{ij} three definitions of distance are met $d_{ij} \in [0,1]$ and d_{ij} indicates the similarity between regions. The greater the similarity, the smaller the distance. This distance definition is used to build a complex network.

1.3. Construction and heterogeneity of complex networks

Taking each region as a node and connecting the edges by the distance between regions, a complex network between regions can be established [25]. The methods for building complex networks include threshold method, visible graph, etc. [26]. In this paper, connecting the four regions with the smallest distance can explain the degree of correlation between regions. The smaller the correlation is, the less transmissibility of air pollution is:

1.3.1. Node degree

In this paper, we first investigate the node degree in the properties of complex networks. The degree of a node refers to the number of other regions directly connected to the node, expressed in K. The greater K is, the closer it is to other regions, the greater the correlation is, and the more likely it is to be a seriously polluted region. It can be seen from **Table 1** that Beijing has the largest share, followed by Suzhou, Langfang, Changsha and Tianshui, followed by Tianjin, Shijiazhuang and other cities.

Table 1. The first 13 areas with larger degrees in complex.

Region	Beijing	Suzhou	Langfang	Changsha	Tianshui	Tian <u>j</u> in	Tangshan
Nodal degree	11	9	8	8	8	7	7
Region	Zhangjiakou	Shijiazhuang	Zhangjiajie	Zhongshan	Xining	Lanzhou	
Nodal degree	7	7	7	7	7	7	

1.3.2. Community classification

The heterogeneity of complex networks is also manifested in the existence of community structure. The network can be divided into multiple groups [27,28]. There are many edges in the group, and the internal structure is dense; however, there are few connecting edges between the group and its internal structure is loose. In general, a group is called a community. The structure of the community forms a hierarchical structure. The network is divided into seven blocks by using Newman Girvan's division method. At this time, the corresponding Q value is the largest, which is 0.655. **Figure 2** shows the community distribution of the complex network, and **Table 2** shows the community classification.

It can be seen from **Figure 2** and **Table 2** that the distribution of these seven communities, of which the bold ones are areas with high degree. Community 1 is central China, community 2 is North China and central China, community 3 is all West China, community 4 is East China, central China, south China and West China, community 5 is all East China, community 6 is East China, south China and West China, and community 7 is East China, north China and West China. It can be seen that air pollution is somewhat related to the region.

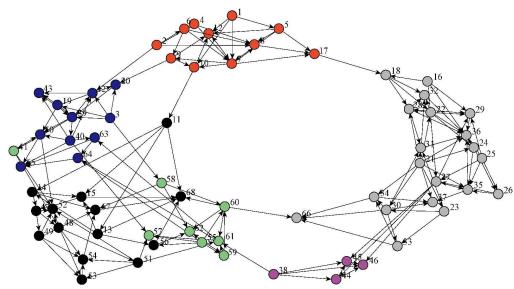


Figure 2. Community distribution of complex network.

Table 2. The classification of community.

Community	Region
1	Chenzhou (41)
2	Yuncheng (38), Zhengzhou (44), Kaifeng (45), Luoyang (46)
3	Xining (55), Urumqi (57), Aksu (58), Turpan (59), Lanzhou (60), Tianshui (61), Lhasa (62)
4	Huangshan (3), Nanchang (19), Jingdezhen (20), Changsha (39), Zhangjiajie (40), Wuhan (42), Jingzhou (43), Guilin (50), Chengdu (63), Mianyang (64), Chongqing (65)
5	Shanghai (1), Hefei (2), Bengbu (4), Nantong (5), Nanjing (6), Suzhou (7), Yangzhou (8), Hangzhou (9), Ningbo (10), Jiaxing (12), Qingdao (17)
6	Wenzhou (11), Fuzhou (13), Xiamen (14), Quanzhou (15), Guangzhou (47), Zhongshan (48), Shenzhen (49), Beihai (51), Nanning (52), Haikou (53), Sanya (54), Hainan (56), Kunming (67), Lijiang (68)
7	Jinan (16), Yantai (18), Beijing (21), Tianjin (22), Xilingol (23), Baoding (24), Hengshui (25), Xingtai (26), Handan (27), Tangshan (28), Cangzhou (29), Zhangjiakou (30), Chengde (31), Qinhuangdao (32), Baotou (33), Hohhot (34), Shijiazhuang (35), Langfang (36), Taiyuan (37), Yinchuan (66)

1.4. Phantom

In order to reveal the structural principle of complex networks, the concept of "motif" is introduced [29–32]. Motif is the basic mode of network construction. The frequency of its recurrence in the network is much higher than that in the random network. In fact, a motif is a large number of small-scale subgraphs with the same structure in the network. This seed graph depicts the specific patterns of interconnection within the network from the local level. Shenorr et al. [33] found various models in biological networks, neural networks, food chains and technical networks, and carried out research on the structural design principle of complex networks. **Figure 3** shows 15 common subgraphs. Taking 3 nodes as an example, there are 13 common motifs.

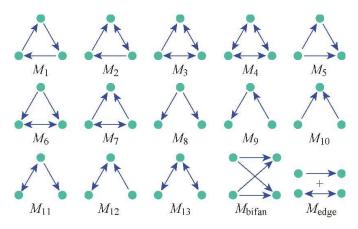
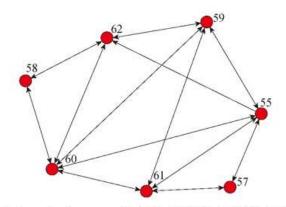
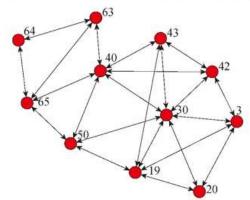


Figure 3. The fifteen major directed sub-graphs.

Use the method of Austin et al. [34] to find 13 seed graphs of 3 nodes in 7 communities of the complex network in five regions. Especially, the subgraphs of regions with large degree are more concerned. Among the 13 seed maps, **Figures 3** and **4** are more distributed. Take **Figure 4** as an example. The subgraph distribution corresponding to **Table 1** is found.

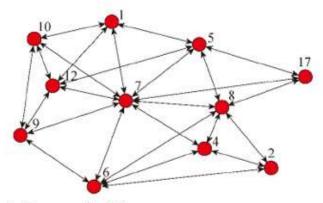


Subgraph of community 3 M₄: 58,60,62; 59,62,60; 59,60,61; 55,60,61; 55,57,61; 55,59,61; 55,60,62; 55,59,60 and so on

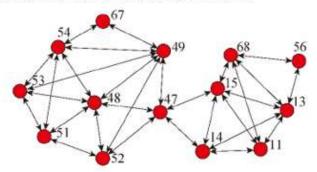


Subgraph of community 4 M4: 63,64,65: 40,63,65; 40,42,43; 39,40,43; 39,42,43; 39,40,42; 40,50,65; 39,40,50; 19,39,43; 3,39,42; 3,19,39; 3,20,39; 3,19,20; 19,20,39 and so on

Figure 4. The distribution of sub-graphs M4 of community 3 and 4.



Subgraph of community 5 M₄: 1,10,12; 9,10,12; 1,5,12; 1,7,12; 7,9,12; 5,7,12; 5,7,17; 5,7,8; 6,7,8; 4,7,8; 7,8,17; 5,8,17; 2,4,8; 2,4,6; 4,6,8; 4,6,7; 1,5,7 and so on



Subgraph of community 6 M₄: 11,13,15; 11,14,15; 13,14,15; 11,13,14; 11,13,68; 14,15,47; 11,15,68; 13,15,68; 13,56,68; 49,54,67; 49,53,54; 51,53,54; 48,53,54; 48,51,53; 48,49,53; 48,40,54; 48,49,52; 47,48,49; 48,51,52; 47,48,52 47,49,52 and so on

Figure 5. The distribution of sub-graphs M4 of community 5, 6 and 7.

2. Results and discussion

68 cities in five regions of China were selected to construct the complex network of air pollution with the correlation size, and the heterogeneity of the corresponding network was studied. Firstly, the size of node degree is analyzed, and 13 regions with large degree are found, including Beijing, Suzhou, Langfang, Changsha, Tianshui, Tianjin, Tangshan, Zhangjiakou, Shijiazhuang, Zhangjiajie, Zhongshan, Xining and Lanzhou. These are areas with relatively serious air pollution.

Then the established network is divided into communities. When n = 7, the Q value is the largest. The 68 districts were divided into 7 communities. The areas in these seven communities are not completely divided according to East China, north China and other regions. For example, community 1 is in Central China, community 3 is all in West China, and community 5 is all in East China. Some communities belong to several regions. For example, community 2 refers to North China and central China, community 4 refers to East China, central China, south China and West China, community 6 refers to East China, south China and West China, and community 7 refers to East China, north China and West China.

Finally, different communities are analyzed by using the method of motif

classification in literature. It is found that M4 and M13 are common. Taking M4 sub motif as an example, communities 3~7, which are more moderate, are searched. In the process of analysis, it was found that the sub motifs in these heavily polluted areas also appeared clustering.

Table 3. The clustering phenomenon of sub-graphs in different communities.

Community	Region
3	Xining (55), Turpan (59), Lanzhou (60), Tianshui (61), Lhasa (62)
4	Huangshan (3), Nanchang (19), Changsha (39), Zhangjiajie (40), Wuhan (42), Jingzhou (43), Guilin (50), Chengdu (63), Chongqing (65)
5	Shanghai (1), Bengbu (4), Nantong (5), Nanjing (6), Suzhou (7), Yangzhou (8), Ningbo (10), Jiaxing (12)
6	Two Clusters: Fuzhou (13), Xiamen (14), Quanzhou (15), Lijiang (68); Zhongshan (48), Shenzhen (49), Beihai (51), Nanning (52), Haikou (53), Sanya (54)
7	Three Clusters: Beijing (21), Tianjin (22), Tangshan (28), Qinhuangdao (32) And Langfang (36); Hengshui (25), Xingtai (26), Handan (27), Shijiazhuang (35); Beijing (21), Xilingol (23), Cangzhou (29), Zhangjiakou (30), Chengde (31), Baotou (33), Hohhot (34)

Table 3 shows the cluster phenomenon of motifs, in which the blackbody part is the area with more connections, indicating that the air pollution in these cities is related. The air pollution index changes according to the clustering phenomenon. This clustering phenomenon should be considered when controlling air pollution. It can prevent the phenomenon that the air pollution in one area is transferred from other relevant areas, and the effect of air pollution control is not obvious. For example, in the process of air pollution control and treatment in Beijing, if these clustered areas can be treated together, the effect of treatment should be better than that of treating Beijing alone. If we want to control air pollution together, we need to work together.

3. Conclusion

In this paper, the complex network method is used to analyze 68 different regions, and the correlation network between air pollution cities is obtained. The properties of the corresponding network are analyzed, including node degree, community structure and motif. In real life, the phenomenon of air pollution is not a single phenomenon in a city, but a phenomenon occurring together in most areas. At the same time, due to the fluidity of the air, it is complex to analyze the problem of air pollution. Through the analysis of the network, the phenomenon of urban agglomeration is found, and it is concluded that the treatment of air pollution needs to treat the relevant cities together, so as to achieve an immediate effect faster.

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