Mathematics and Systems Science 2025, 3(2), 3120.
https://doi.org/10.54517/mss3120

Article

Inherit variability characterization considering piezocone test data: Application to bauxite tailings

Juliano Pasa de Campos, André Luis Meier, Arthur Felisbino, Gracieli Dienstmann*

Federal University of Santa Catarina, 205 João Pio Duarte da Silva St., Florianópolis 88040-900, Santa Catarina, Brazil
*** Corresponding author:** Gracieli Dienstmann, g.dienstmann@ufsc.br

CITATION

de Campos JP, Meier AL, Felisbino A, Dienstmann G. Inherit variability characterization considering piezocone test data: Application to bauxite tailings. Mathematics and Systems Science. 2025; 3(2): 3120. https://doi.org/10.54517/mss3120

ARTICLE INFO

Received: 2 December 2024
Accepted: 6 May 2025
Available online: 10 June 2025

Abstract: Variability characterization is a key component in the reliability assessment of geotechnical systems, particularly in scenarios involving tailings deposits, where spatial heterogeneity can critically impact design safety. Despite advances in modeling spatial randomness, many reported cases still rely on theoretical assumptions to define appropriate statistical characterizations—such as theoretical probability density functions (PDFs) and correlation structures—which may misrepresent site-specific conditions. To bridge this gap, piezocone penetration tests (CPTu) stand out as promising tools for providing continuous measurements along a vertical profile that can be used to define statistical behavior and avoid bias. This paper presents a spatial variability characterization of a bauxite tailings deposit based on mechanical parameters derived from CPTu data. The study includes basic statistical analysis—mean (μ), standard deviation (σ), coefficient of variation (CV)—alongside a comparison with theoretical PDFs. Subsequently, spatial correlation is evaluated through covariance analysis and estimation of the vertical scale of fluctuation (δ), using a dedicated subroutine that fits theoretical autocorrelation models (TAMs). The deposit is classified as highly variable according to the I_{cRW} index. The normal and Weibull PDFs best represent the data distributions. The vertical scales of fluctuation vary significantly: 0.01 m to 4.43 m for cone resistance (q_c), 0.01 m to 4.36 m for sleeve friction (f_s), and 0.01 m to 5.00 m for pore water pressure (u_2). These findings offer valuable input for probabilistic stability and serviceability analyses, contributing to safer and more informed geotechnical designs involving mine tailings.

Keywords: spatial variability; in situ tests; autocorrelation models; piezocone test; bauxite tailings; geotechnical engineering

1. Introduction

The difficulties associated with the construction and operation of Tailings Storage Facilities (TSFs) have long been recognized and remain a significant concern for the engineering community. In Brazil, there has been increasing significance in dam failure over the last decade, especially due to the occurrence of two recent failures (Brumadinho and Mariana Dams) with notable socioenvironmental impacts.

Central to the design of TSFs is the correct definition of geotechnical engineering parameters that arises from the heterogeneous nature of waste products, hydraulic depositional processes, and behavior changes during the lifetime of deposits. These production and deposition processes generate a material that is extremely variable both vertically and horizontally [1], making spatial variability a critical factor influencing stability and drainage behavior.

Bauxite tailings fit within this context due to their formation processes, which produce fine-grained particles and heterogeneous depositional patterns, leading to significant differences in strength, compressibility, and permeability along both vertical and horizontal directions. Such variability critically affects pore pressure dissipation and shear strength evolution, key aspects for the stability and long-term performance of TSFs. Nevertheless, specific investigations into the spatial variability of bauxite tailings remain limited in the literature, highlighting the motivation for the present study.

Variability characterization is an essential tool in reliability analysis, but despite the progress made in implementing routines that facilitate the modeling of random soil properties, such as the Random Field Model by Fenton and Griffiths [2], many reported cases still rely on theoretical assumptions to define statistical characterizations, such as theoretical probability density functions (PDFs) and correlation structures. To address these data limitations, piezocone tests, which provide continuous measurements along a vertical profile, offer a substantial dataset that enables the establishment of reliable relationships and minimizes potential biases in statistical and probabilistic applications [3]. More recently, in Dienstmann et al. [4], piezocone test data from a gold tailings deposit were considered in a statistical analysis to address the influence of inheritance variability on drainage behavior.

Within this context, the present paper statistically characterizes key parameters, including the mean (μ), coefficient of variation (CV), fluctuation scales (δ), and Probability Density Functions (PDFs), by analyzing piezocone data soundings. A series of tests conducted on a bauxite mine tailings storage facility (TSF) is considered in this study, which evaluates direct measurements of tip resistance q_c, friction sleeve f_s, and pore pressure u_2 and their residual values (after trend removal). Focusing on the correlation structure, an algorithm was developed to define, in an optimized way, the best fit correlation between the theoretical autocorrelation model (TAM) and the sample autocorrelation itself, providing direct and fast evaluation of vertical fluctuation scales.

2. Spatial variability

The application of statistical methods to describe the variability in a dataset is more straightforward when the data exhibit a certain degree of predictability. When extended to the characterization of spatial variability in geotechnical materials, this concept translates into the recommendation to apply such methods only within homogeneous mechanical behavior layers, which in most cases are formed from the same base process. Within homogeneous layers, trends in material parameter variations become evident, allowing the application of the decomposition method mentioned by Campello et al. [5]. This method involves the idealization that the “true” value of a geotechnical property ($\xi(z)$) along a depth z can be described in terms of a trend function ($t(z)$) and a fluctuating component ($w(z)$), as per Equation (1):

$$\xi(z) = t(z) + w(z) \tag{1}$$

Phoon et al. [6] exemplify, through an illustration, the behavior described by the equation above. **Figure 1** presents a typical pattern of variability in a homogeneous layer profile.

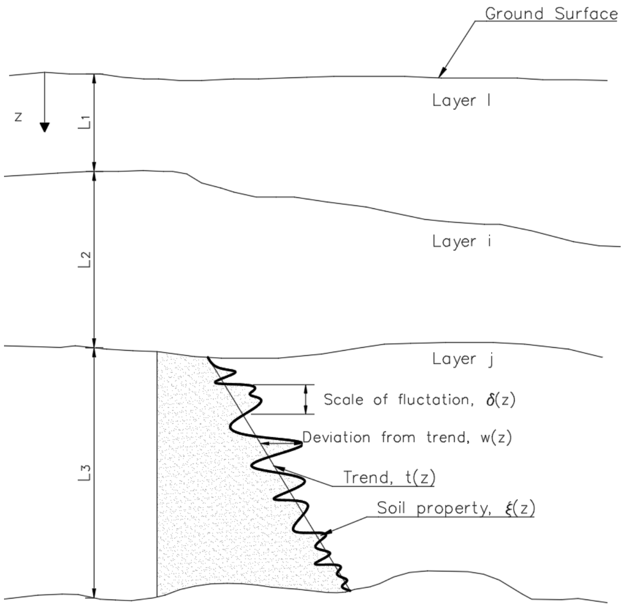


Figure 1. Inherent soil variability (adapted from Phoon et al. [6]).

Uzielli et al. [7] asserted that choosing the descriptive trend function is a complex task and impacts both the correlation structure and the statistical parameters describing the random field. The initial step in characterizing the variability of geotechnical parameters is to determine the best polynomial function that describes the data trend. Salgado et al. [3] recommend starting attempts with a first-order polynomial, and the regression coefficient (R) should not be less than 0.85. Otherwise, higher degrees of polynomials should be tested. The removal of the trend is an important step since, after this, it is possible to access and analyze the residual component separately.

In **Figure 2**, two additional parameters used to describe the behavior of residuals around the trend function are illustrated. Coefficient of variation (CV) and fluctuation scale (δ). The CV is a fundamental parameter that indicates the extent of the dispersion of residuals, whereas the δ parameter is an indicator of the frequency of residual oscillations.

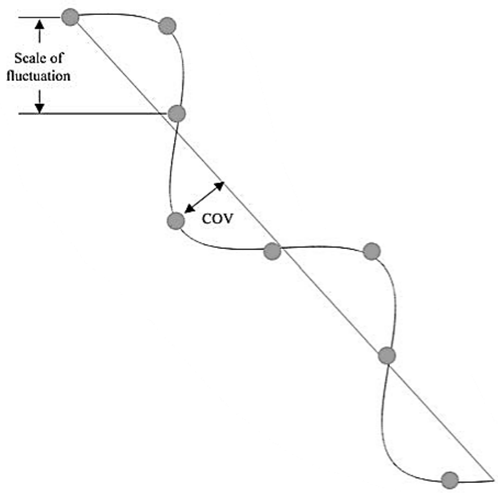


Figure 2. Residual behavior around the trend (Salgado et al. [3]).

The CV is a fundamental parameter that indicates the extent of the dispersion of residuals, whereas the δ parameter is an indicator of the frequency of residual oscillations. The CV can be obtained through the ratio of the standard deviation to the mean, as per Equation (2):

$$\mathrm{CV} = \sigma/\mu \tag{2}$$

The fluctuation scale is the distance between two soil parcels where their parameters exhibit a strong correlation. This parameter can be calculated through different methods; however, one of the most commonly used methods is the method of fitting theoretical autocorrelation models (TAMs) to the sample autocorrelation function. The aim is to find the theoretical function that best represents the sample [8].

The sample autocorrelation function (ρ), (Equation (3)), is derived from the covariance (C) between the separation distance (τ) and the desired parameter. The covariance is obtained via Equation (4), and the separation distance is obtained via Equation (5) [3]:

$$\rho\left(\tau_j\right) = \frac{C_{(\tau_j)}}{C_{(\tau_1)}} \tag{3}$$

$$C\left(\tau_j\right) = \frac{1}{n}\sum_{i=1}^{n-j+1}\left(x_i - \mu_X\right)\left(x_{i+j-1} - \mu_X\right) \tag{4}$$

$$\tau_j = (j-1)\Delta_z \tag{5}$$

where:

μ_X is the mean value of X.

$j \geq 1$ is a whole number associated with the separation distance.

Δ_z is the minimum distance between two consecutive points.

$C(\tau_1)$ is the covariance for the null separation distance.

The theoretical functions are mathematical models, with the most commonly used geotechnical models listed in **Table 1**, namely, the exponential, second-order Markov, and exponential cosine models. The theoretical autocorrelation models should be directly compared with the experimental values calculated via Equation (3).

Table 1. Autocorrelation models.

TAM	**Autocorrelation function**
Exponential	$\rho(\tau) = \exp\left(\frac{-2\lvert\tau\rvert}{\delta}\right)$
Second Order Markov	$\rho(\tau) = \left(1 + 4\frac{\lvert\tau\rvert}{\delta}\right) exp\left(-4\frac{\lvert\tau\rvert}{\delta}\right)$
Squared Exponential	$\rho(\tau) = exp\left[-\pi\left(\frac{\lvert\tau\rvert}{\delta}\right)^2\right]$
Exponential Cosine	$\rho(\tau) = exp\left(-\frac{\lvert\tau\rvert}{\delta}\right) cos\left(\frac{\lvert\tau\rvert}{\delta}\right)$

where [1] ρ is the autocorrelation function, τ is the separation distance and δ is the scale of fluctuation.

Extending the concepts described above to tailings implies careful consideration of trend variability. Direct comparisons of piezocone tests considering different investigation islands from the same TSF can be categorized into distinct behaviors, as reported in Dienstmann et al. [9]. The regional behavior of coarse and fine materials is a result of the depositional process, in which coarse particles settling from the slurry are transported along the beach by saltation and rolling, whereas the finer suspended or colloidal particles settle only when they reach the still water of the decant to form slime zones, as suggested by Vick [10]. In this regard, the data presented in this study focus on highly variable behavior materials because of the heterogeneous grain distribution along the deposition area. Both direct parameters and those after trend removal are evaluated. The trend removal considers eliminating from the statistical analysis the expected variation in parameters such as q_c and u_2, mainly with depth. The obtained results should be considered as a possible range for future applications.

3. Materials and methods

3.1. Piezocone test data

For in situ characterization of TSFs, the piezocone test (CPTu) is the most widely used testing instrument [11]. The piezocone test involves driving a conical tip into the soil at a standardized rate according to ASTM D5778-07 [12]. During penetration, load sensors and pressure transducers measure the cone resistance (q_c), sleeve friction (f_s), and pore pressure (u_2) with depth. The basic measurements of the test (q_c, f_s and u_2) are used for material behavior characterization. **Figure 3** presents typical profiles from tests conducted in the bauxite tailings deposit considered in this research. Despite the fluctuations in the behavior parameters, there is a general trend of increasing values, particularly for the cone resistance q_c and pore pressure u_2, with depth. The figure also shows the behavior index I_{cRW} of Robertson and Wride [13]. The index is used to classify the typical behavior of a material in terms of strength and drainage. In this context, I_{cRW} values greater than 2.95 are attributed to materials with typically undrained behavior (fine materials, clays), whereas I_{cRW} values less than 2.6 correspond to materials with drained behavior (coarse materials, sands). Finally, intermediate values, $2.6 < I_{cRW} < 2.95$, indicate materials with intermediate behavior (silts).

The dataset presented herein comprises CPTu soundings performed between 1999 and 2005 as part of the ALUMAR ARB tailings characterization campaign. The complete dataset is documented in Bedin [14]. In the present study, only fine-grained materials exhibiting undrained behavior were analyzed; for this purpose, profiles were selected based on the criterion of $I_{cRW} > 2.95$. Although the filtering process primarily relied on the I_{cRW} classification, a preliminary visual inspection of the profiles was also performed to identify and exclude inconsistent or anomalous data, such as equipment noise or incomplete soundings. Specifically, **Figure 3** shows piezocone profiles from the 2005 field test campaign, comprising cone tip resistance q_c, friction sleeve f_s, and pore pressure u_2, along with the considered classification parameter I_{cRW}. Notably, in this particular campaign, predominantly fine material was characterized by low values of q_c and higher pore pressures u_2 combined with I_{cRW} greater than 2.95.

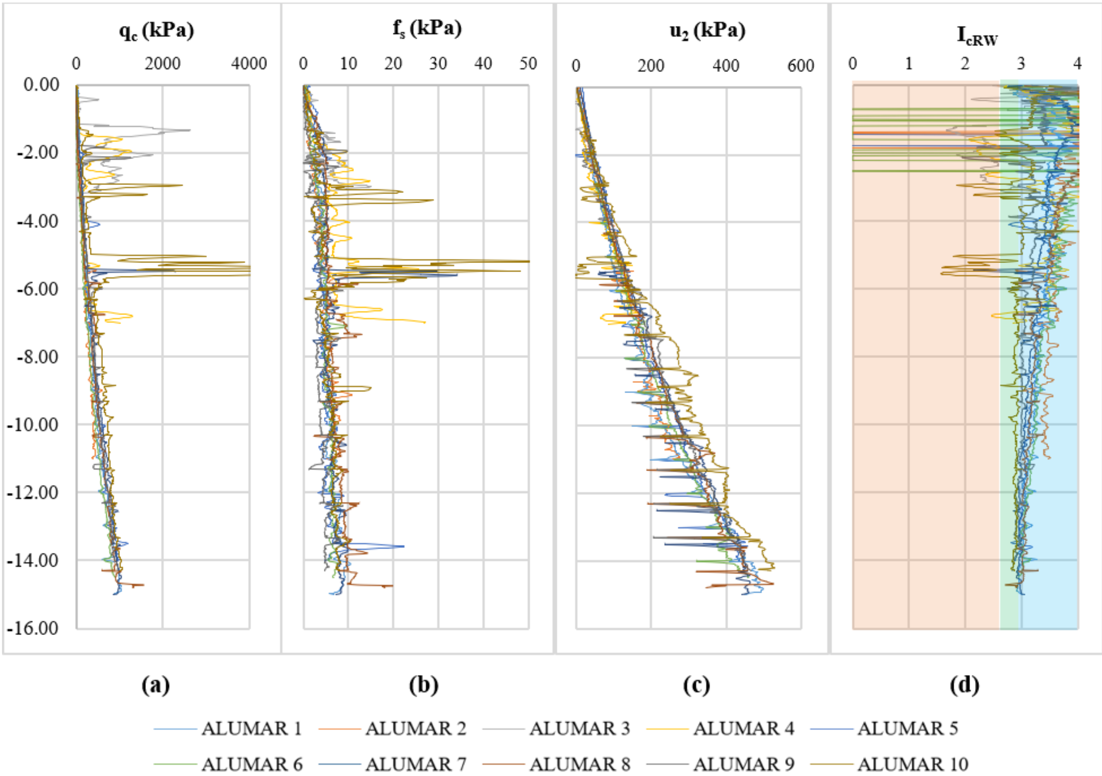


Figure 3. Results from the piezocone test of ARB#3 performed in the 2005 campaign. Where **(a)** are the q_c profiles of ALUMAR 1–10; **(b)** are the f_s profiles of ALUMAR 1–10; **(c)** are the u_2 profiles of ALUMAR 1–10; **(d)** are the I_{cRW} classifications of ALUMAR 1–10.

3.2. Variability characterization (method)

The algorithm developed for this research reads CPTu data from an Excel sheet and executes a behavior selection procedure, dividing the sample according to its I_{cRW} classification into coarse (drained behavior), intermediate (partially drained behavior), and fine soil (undrained behavior) categories. Afterwards, basic statistical analysis and spatial variability characterization are performed.

Prior to the statistical evaluation, trend inspection was conducted. **Figure 4a** depicts the definitions of the trend and residual values, alongside the measured data. The average tip resistance (q_c) clearly increases with depth (z). To address this trend, a linear function (e.g., $q_c = az + b$, where 'a' and 'b' are fitting parameters) was applied to fit the CPTu data. After removing the trend component ($az + b$) from the original CPTu data, the residual tip resistance forms a stationary random field with a zero mean, as commonly used in random field theory for stationary fields [2]. The stationarity verification was performed through the inspection of the calculated residual fields. Stationary fields are expected to exhibit a zero mean and a normal distribution (normality). An example of the normality assessment is presented in **Figure 4b**. This analysis of stationarity is important for the accurate calculation of the fluctuation scale.

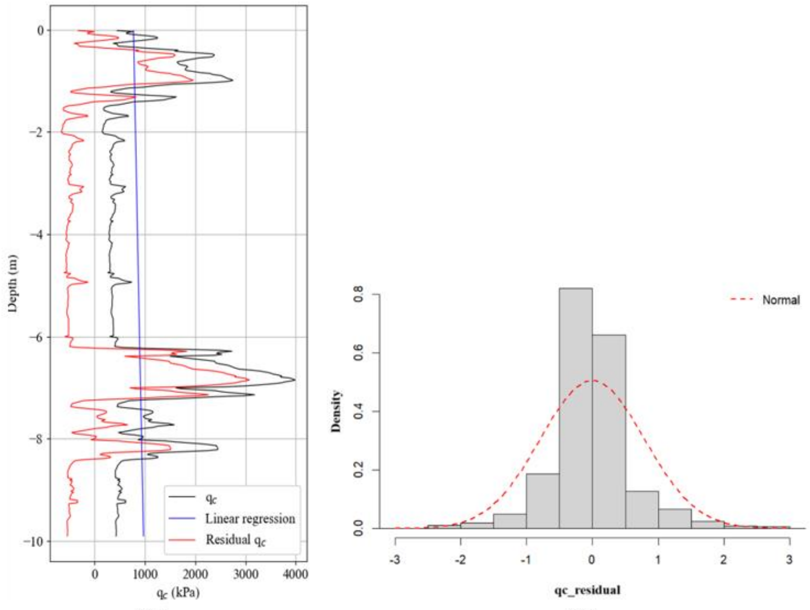


Figure 4. Variability pre-treatment: **(a)** general profile and residual plot along depth for ALUMAR 3; **(b)** residual q_c normality evaluation.

In addition to the stationary evaluation, the basic statistical analysis consists of computing the mean (μ), standard deviation (σ), and coefficient of variation (CV). Furthermore, histograms of the CPTu parameters were generated, and comparisons between theoretical probability density functions (PDFs) and empirical data were performed using QQ plots. QQ plots were constructed by plotting theoretical quantiles against empirical quantiles and serve to assess the goodness of fit between the observed data and the selected theoretical distribution. A good fit is indicated when the points approximately form a straight line. The PDFs tested in this research are the normal, log-normal, exponential, gamma, and Weibull distributions, all commonly applied in geotechnical engineering analyses.

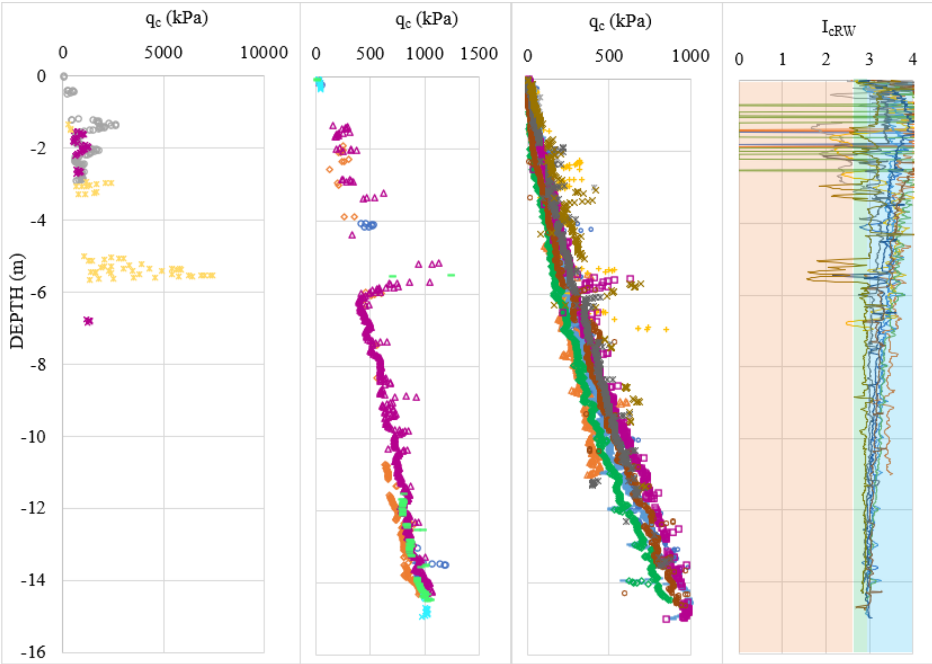
In the spatial variability step, the sample autocorrelation function was computed considering the covariance function, as defined by equations 3 to 5. This sample autocorrelation was then compared with the theoretical autocorrelation models (TAMs) listed in **Table 1**. Since the TAMs are functions of the scale of fluctuation, this parameter was adjusted by minimizing the distance between each point of the theoretical curves and the corresponding points of the sample autocorrelation curve. The final scale of fluctuation adopted corresponds to that of the theoretical model that achieved the highest coefficient of determination (R^2) when compared with the sample autocorrelation function.

4. Results and discussion

4.1. Basic statistics and probability density functions

As mentioned in the methods section, before any statistical or variability analysis, a mechanical behavior selection of the layers in the field test results was considered. **Figure 5** illustrates the selection of layers and reading according to the results of the I_{cRW}. In this sense, **Figure 5a** displays only q_c readings in layers where $I_{cRW} < 2.60$

(coarse soil); **Figure 5b** shows the q_c readings in layers where $2.60 < I_{cRW} < 2.95$ (intermediate soil); and **Figure 5c** shows the q_c readings in layers where $I_{cRW} > 2.95$ (fine soil). A layer was considered representative of only part of the analysis if it consisted of at least 10 readings, considering the sensitivity of the equipment, which is directly associated with its diameter.

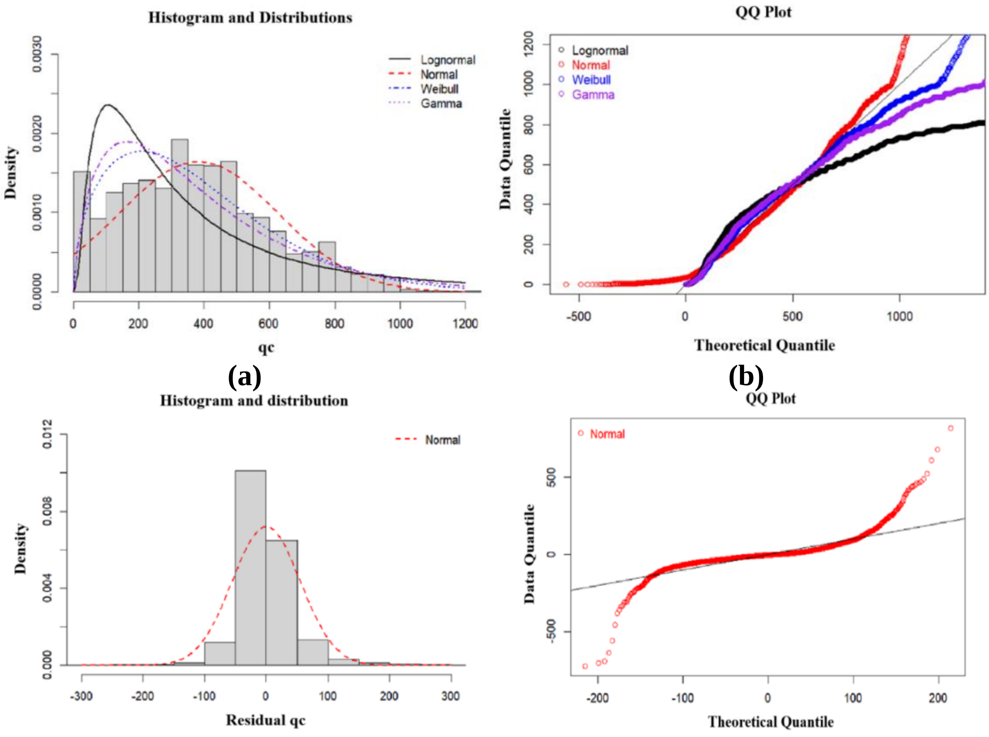


(a) **(b)** **(c)** **(d)**

Figure 5. Results of the cone tip resistance, separated by the I_{cRW} index for the piezocone field tests at ARB#3 performed in the 2005 campaign. Where **(a)** are the q_c readings in layers where $I_{cRW} < 2.60$ (coarse soil); **(b)** are the q_c readings in layers where $2.60 < I_{cRW} < 2.95$ (intermediate soil); **(c)** are the q_c readings in layers where $I_{cRW} > 2.95$ (fine soil); and **(d)** are the I_{cRW} classifications along the sounding profiles of ARB#3 performed in the 2005 campaign.

For the clayey behavior, the distributions found for the main analyzed parameters are presented below, along with the fitting analysis with theoretical distributions via the Q–Q plot.

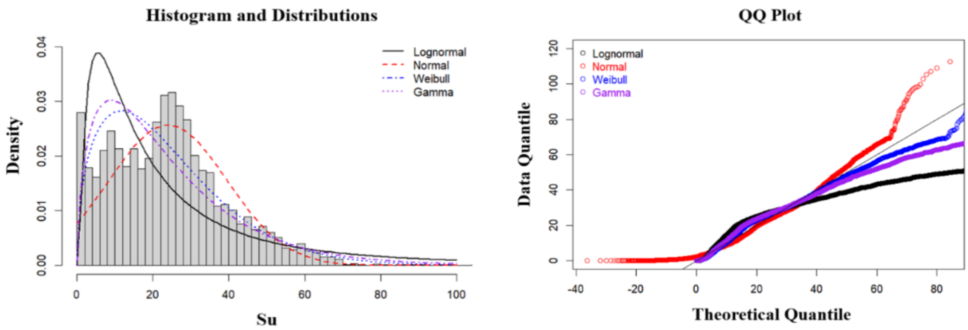
The histogram distribution of the parameter q_c in **Figure 6** shows that the values range from 0 kPa to 1000 kPa. When the parameter distribution is compared with the theoretical curves, the best fit is the normal distribution, as confirmed by the Q–Q plot. For the residuals, a variation from −200 kPa to 200 kPa is observed, indicating a good fit to the normal distribution. The analysis of the normality of the residuals from the readings is a fundamental step in defining the homogeneity/stationarity of the profile. Random probabilistic methods, such as the Local Average Subdivision (LAS) method by Fenton and Griffiths [2], rely on this basic premise for predicting behavior.



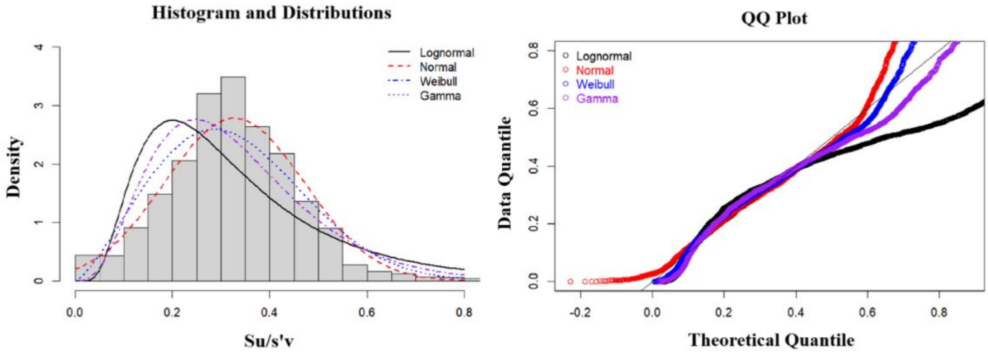
(a) (b) (c) (d)

Figure 6. Results from the statistical analysis of the grouped fine soil of the 2005 campaign where **(a)** is the histogram distribution of q_c; **(b)** is the QQ plot of the theoretical distributions fitting the sample data; **(c)** is the histogram distribution of residual q_c; **(d)** is the QQ plot of the normal distribution fitting the sample residual data.

For the parameter S_u, as shown in **Figure 7**, variations from 0 kPa to approximately 70 kPa are observed in the histogram distribution of the parameter. Among the theoretical distribution curves tested, the best fits are the Weibull and normal distributions, as indicated by the Q–Q plot. For the parameter normalized by vertical stress, the values are distributed between 0 kPa and 0.8 kPa, following a curve very close to a normal distribution, as shown in the Q–Q plot results.



(a) (b)



(c) **(d)**

Figure 7. Results from the statistical analysis of the grouped fine soil of the 2005 campaign. where **(a)** is the histogram distribution of Su; **(b)** is the QQ plot of the theoretical distributions fitted to the sample data; **(c)** is the histogram distribution of Su normalized by the effective vertical stress; and **(d)** is the QQ plot of the theoretical distributions fitted to the sample normalized data.

Table 2 presents a summary of the basic statistics applied to the grouped parameters, showing the mean values, standard deviations, and coefficients of variation for the clayey behavior. The results indicate that the coefficients of variation range from 0.46 (46%) to 1.05 (105%), with the lowest coefficient of variation for the normalized undrained shear strength parameter relative to the effective vertical stress. The normalization of undrained shear strength by effective stress is a commonly used approach in geotechnics (e.g., the SHANSEP method, Ladd and Foott [15]). It allows for analyzing the behavioral characteristics of the material and extrapolating Su values across the entire deposit if the acting stresses are known. Additionally, normalization by effective stress can be interpreted as a form of preprocessing of the profiles, as suggested by Milan and Dienstmann [16]. For the undrained shear strength data evaluated in this study, the normalization approach resulted in a better approximation of the data to a normal distribution.

Table 2. Basic statistics applied to the grouped parameters of fine soil from the 2002, 2004, and 2005 campaigns.

Parameter	μ	σ	**CV**	**PDF**
q_c	382.84 kPa	243.05 kPa	63%	NORMAL
f_s	6.95 kPa	7.31 kPa	105%	WEIBULL
u_2	190.94 kPa	135.26 kPa	71%	NORMAL
S_u	23.70 kPa	15.70 kPa	66%	WEIBULL/NORMAL
S_u/σ'_{vt}	0.32	0.15	46%	NORMAL

Becker et al. [17] recently presented statistical insights into the variability in the undrained shear strength of iron tailings from the Germano dam in Mariana. Their study carefully separated data from multiple profiles and evaluated the distribution of the normalized undrained shear strength. The analysis focused exclusively on layers exhibiting plastic behavior and considered the impact of layer thickness. The findings revealed that the S_u/σ'_{vt} ratio for the plastic tailings followed a lognormal distribution. Moreover, thicker layers of plastic tailings were associated with lower values and reduced variability. The ratio S_u/σ'_{vt} was found to range between 0.11 and 0.24, with coefficients of variation (CVs) ranging from 29% to 47%. These results generally indicate that the analyzed bauxite tailings exhibit greater normalized strength, with a mean S_u/σ'_{vt} ratio of 0.32 and a CV of 46%.

4.2. Scale of fluctuation

The calculation of scales of fluctuation performed by the algorithm involves approximating theoretical autocorrelation models to the sample's autocorrelation curve. The best approximation result for each analyzed case corresponds to the determined fluctuation scale. **Figure 8** shows a graphical example of applying this method to the homogeneous layer between −0.34 m and −14.24 m in borehole Alumar 05 from the 2005 campaign in ARB#3.

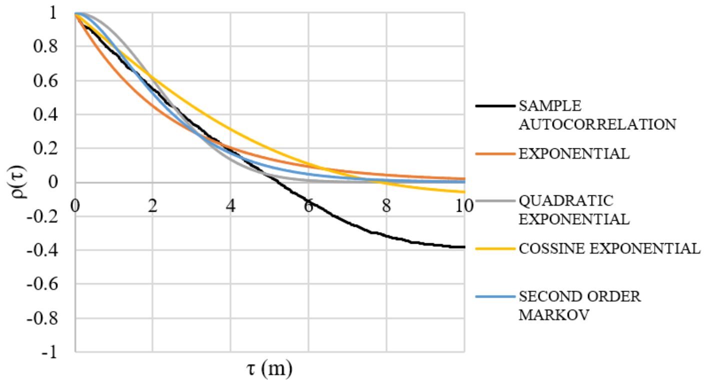


Figure 8. Theoretical models adjust to the sample autocorrelation curve.

For each layer and each observed behavior, the scale of fluctuation and basic statistical parameters were calculated, as illustrated in **Table 3**, a typical example. As shown in the example table, the locally analyzed layers are listed in each row, where the "ID" column serves as the identifier, classifying them by year, sounding station, and position relative to the sounding depth. There is no exact pattern regarding the depths where the layers corresponding to each behavior are located.

Table 3. Local variability considering q_c readings in layers of fine soil mechanical behavior.

ID	μ (kPa)	σ (kPa)	CV	δ_v(m)	Model δ	Thickness (m)
(2002) Alumar01 2.90–3.55	287.62	37.52	0.13	0.35	Exponential	0.66
(2002) Alumar01 4.8–5.02	444.55	136.74	0.31	Ø	Quadratic Exponential	0.22
(2002) Alumar01 5.20–5.38	475.87	122.90	0.26	Ø	Cosine Exponential	0.18
(2002) Alumar01 5.62–5.92	435.84	65.51	0.15	Ø	Exponential	0.3
(2002) Alumar01 7.5–7.82	612.15	129.92	0.21	Ø	Exponential	0.32
(2004) Alumar01 4.3–4.52	485.25	35.65	0.07	0.09	Cosine Exponential	0.22
(2004) Alumar01 7.08–7.26	501.40	8.25	0.02	Ø	Cosine Exponential	0.18
(2004) Alumar01 7.34–7.96	484.59	17.64	0.04	0.52	Cosine Exponential	0.62
(2004) Alumar01 8.24–9.28	525.85	57.76	0.11	0.11	Cosine Exponential	1.04
(2002) Alumar01 2.90–3.55	601.05	115.21	0.19	4.32	Cosine Exponential	6.52
(2002) Alumar01 4.8–5.02	12.62	11.05	0.88	0.06	Second Order Markov	0.18

Due to the large extension of the data and for simplification, a summary table is shown below. **Table 4** presents the range of the results found for the local variability parameters of the campaigns performed in 2002, 2004, and 2005 for the fine soil (clayey behavior).

For the parameter q_c, the mean values range from 12.62 kPa to 1304.70 kPa, the standard deviation from 0 kPa to 442.82 kPa, the coefficient of variation from 0.00 to 1.40, and the fluctuation scale from 0.01 m to 4.43 m. For the parameter f_s, the mean values range from 0.00 kPa to 68.90 kPa, the standard deviation from 0.00 kPa to 32.21 kPa, the coefficient of variation from 0.00 to 1.30, and the scale of fluctuation from 0.01 m to 4.36 m. The mean values of u_2 range from −3.24 kPa to 525.10 kPa, the standard deviation from 0.00 kPa to 176.66 kPa, the coefficient of variation from −1.06 to 4.00, and the scale of fluctuation from 0.01 m to 5.00 m.

Table 4. Range of results found for the local variability in fine soils of the campaigns performed in 2002, 2004, and 2005.

Para-meter	μ (kPa)	σ (kPa)	CV	δ (m)	Thickness (m)
q_c	12.62 to 1304.70	0.00 to 442.82	0.00 to 1.40	0.01 to 4.43	0.15 to 14.46
f_s	0.00 to 68.90	0.00 to 32.21	0.00 to 1.30	0.01 to 4.36	0.15 to 14.46
u_2	−3.24 to 525.10	0.00 to 176.66	−1.06 to 4.00	0.01 to 5.00	0.15 to 14.46

5. Conclusion

This study conducted detailed analyses of statistical characterization and spatial variability via bauxite mine tailings from ALUMAR in São Luís, Maranhão, considering CPTu tests conducted in 2002, 2004, and 2005. The CPTu test campaigns provided essential data to determine the mechanical and spatial variability parameters of the material, enabling nearly continuous readings along the sounding profiles.

Preliminary analysis via spreadsheets derived parameters from piezocone tests to characterize the material's mechanical behavior for each probe reading, revealing high variability. This prompted the need to separate homogeneous layers for accurate statistical analysis. The homogeneous layers were subsequently identified and classified into three distinct behaviors (sandy, silty, and clayey) via a VBA algorithm. The ARB profile was found to consist of overlapping lenses with differing behaviors. The spatial and statistical variability of these layers was characterized via a Python script, which revealed that a normal distribution best fits most normalized parameter residuals. Basic test values, along with parameters Su and Su/σ'_v (for clayey behavior), were grouped to represent the global variability of the bauxite residue. The mean values, standard deviations, and coefficients of variation were calculated for each layer, and the global statistics were grouped. Typical and representative results for these materials remain scarce in the literature.

These comprehensive analyses allowed for a detailed and robust evaluation of the variability of mechanical parameters, offering critical insights and essential data for conducting a reliability analysis that more accurately reflects the actual conditions of the tailings. This level of precision contributes significantly to understanding the behavior of the material under real-world conditions, supporting more reliable and informed decision-making in the management and assessment of the structural stability of tailings.

However, it is important to note that the results presented in this study are specific to the deposit analyzed and should be interpreted with caution. The findings aim to contribute to a broader understanding of the inherent variability of geotechnical materials but may not be directly generalizable to other deposits. Therefore, it is recommended that each site be evaluated individually and the data presented herein be used primarily as reference values rather than definitive parameters for different deposits.

Author contributions: Conceptualization, JPdC and GD; methodology, JPdC and GD; software, JPdC, AF and GD; validation, JPdC, ALM and GD; formal analysis, JPdC, ALM and GD; investigation, JPdC; resources, GD; data curation, GD; writing—original draft preparation, JPdC and GD; writing—review and editing, JPdC and GD; visualization, JPdC and GD; supervision, GD; project administration, GD; funding acquisition, GD. All authors have read and agreed to the published version of the manuscript.

Institutional review board statement: Not applicable.

Informed consent statement: Not applicable.

Conflict of interest: The authors declare no conflict of interest.

References

1. Schnaid F, Dienstmann G, Odebrecht E, et al. A simplified approach to normalisation of piezocone penetration rate effects. Géotechnique. 2020; 70(7): 630-635. doi: 10.1680/jgeot.18.t.033
2. Fenton AG, Griffiths DV. Risk assessment in geotechnical engineering. John Wiley and Sons; 2008.
3. Salgado R, Prezzi M, Ganju E. Assessment of Site Variability from Analysis of Cone Penetration Test Data. Purdue University; 2015. doi: 10.5703/1288284315523
4. Dienstmann G, Perini L, Meier AL, et al. Incorporating inherited variability into drainage effect analysis of piezocone tests in gold tailings. In: Proceedings of the Institution of Civil Engineers—Geotechnical Engineering; 2025. doi: 10.1680/jgeen.23.00082
5. Campello IC, Gardoni MG, Pimentel KCA, et al. Characterization of Vertical Spatial Variability of Soils Using CPTu Data Exploration. In: Proceeding of the 8th International Symposium for Geotechnical Safety & Risk (ISGSR 2022); 2022. doi: 10.3850/978-981-18-5182-7_00-03-010.xml
6. Phoon K, Kulhawy FH, Grigoriu MD. Reliability-Based Design of Foundations for Transmission Line Structures. Cornell University, Ithaca, New York: Eletric Power Research Institute; 1995.
7. Uzielli M, Vannucchi G, Phoon KK. Random field characterisation of stress-nomalised cone penetration testing parameters. Géotechnique. 2005; 55(1): 3-20. doi: 10.1680/geot.2005.55.1.3
8. Kenarsari EA, Chenari RJ, Eslami A. Characterization of the correlation structure of residual CPT profiles in sand deposits. International Journal of Civil Engineering. 2013.
9. Dienstmann G, Schnaid F, Maghous S, et al. Piezocone Penetration Rate Effects in Transient Gold Tailings. Journal of Geotechnical and Geoenvinronmental Engineering. 2018.
10. Vick SG. Planning, design, and analysis of tailings dams, 2nd ed. BiTech, Vancouver, Canada; 1990.
11. Ayala J, Fourie A, Reid D. Improved cone penetration test predictions of the state parameter of loose mine tailings. Canadian Geotechnical Journal. 2022; 59(11): 1969-1980. doi: 10.1139/cgj-2021-0460
12. American society for testing and materials. D5778: Standard Test Method for Electronic Friction Cone and Piezocone Penetration Testing of Soils. West Conshohocken; 2007.

13. Robertson PK, Wride CE. Evaluating cyclic liquefaction potential using the cone penetration test. Canadian Geotechnical Journal. 1998; 35(3): 442-459. doi: 10.1139/t98-017
14. Bedin J. Interpretation of piezocone tests on bauxite residues (Portuguese). Universidade Federal do Rio Grande do Sul; 2006.
15. Ladd CC, Foott R. New Design Procedure for Stability of Soft Clays. Journal of the Geotechnical Engineering Division. 1974; 100(7): 763-786. doi: 10.1061/ajgeb6.0000066
16. Milan H, Dienstmann G. Assessment of vertical spatial variability in a marine sedimentary deposit based on standard penetration tests. Marine Georesources & Geotechnology; 2025. doi: 10.1080/1064119x.2025.2481494
17. Becker LB, Aguiar ALS, Lacerda WA. Variability of the undrained strength of the plastic tailings from the Germano dam in Mariana, Brazil. Geomechanics and Geophysics for Geo-Energy and Geo-Resources; 2024. doi: 10.1007/s40948-024-00750-4