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Brief Report

The new concept of ternary logic and the problems of its implementation

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Abstract: The article deals with the problem of ternary logic, in which, except for the states "yes" and "no" (inherent in the traditional binary logic), is introduced the "third state" U—"Unknown", and also addresses the issues of technical and mathematical problems that arise in this case. To do this, the ternary logic of Stephen Kleene has been corrected, and the implementation of ternary calculations using probabilistic polynomials over the field of real numbers has been proposed. The authors analyze the process of the addition of the "third state" to binary logic, in that regard the approach of Stephen Kleene, who introduced an "indefinite state". But the authors judge it necessary to introduce also the negation of the "indefinite state" and suggest a way to replace logical functions with probabilistic polynomials based on the field of real numbers, which are conveniently calculated on modern hardware, for example, in video card processors. Terms of the ternary logic can be useful for the implementation of new artificial intelligence projects that model the operation of thinking with uncertain results, while the transition to probabilistic functions can expand the capabilities of such models and simplify the analysis of errors that occur during the operation of artificial intelligence systems.

Keywords: logic; mathematical logic; polynomial; real numbers; probability; Boolean functions; models; artificial intelligence; artificial consciousness

1. Introduction

Around 1910 [1] Charles Sanders Peirce defined a many-valued logic system, but he never published it. In fact, he did not even number the three pages of notes where he defined his three-valued operators. Peirce soundly rejected the concept that all the propositions must be either true or false. However, despite his conviction that "the Triadic Logic is universally true", Peirce also jotted down that "all this is mighty close to nonsense." Only in 1966, when Max Fisch and Atwell Turquette started to publish the concepts that they had rediscovered in his unedited manuscripts, did the main points of triadic logic of Priece become widely known.

Generally, the primary motivation for research of three-valued logic is to define the correct value of a statement that cannot be represented as true or false. Initially, Łukasiewicz developed three-valued logic in connection with the problem of future contingents to represent the true value of statements concerning the undetermined future [2]. Bruno de Finetti used the "third value" to formulate the cases when "the given particular individual does not know the [correct] response, at least at a given moment." Hilary Putnam used the same logic to represent the values that cannot be decided physically.

For example, if we have fixed (using a speedometer) velocity of a motor-car, it might be impossible in these circumstances to verify or falsify certain statements concerning its position at the same point in time. If we know, by referring to the laws of physics in combination with certain observed data, that a statement related to the position of the automobile can never be falsified or verified, we can't regard this statement as true or false, but we must define it as "middle". It is only because in macrocosmic experience everything that we regard as an empirically meaningful statement seems to be at least potentially verifiable or falsifiable, because we hold the view that any such statement is either true or false, but in many cases we don't know whether it is true or it is false.

Similarly, Stephen Cole Kleene used a "third value" to represent predicates that are "undecidable by [any] algorithms whether true or false" [3].

The task of modeling and reproducing human consciousness and the implementation of artificial intelligence constantly faces the limitations of the binary logic, which is based on only two states of "yes" and "no", that is, on the truth of a statement or on its falsity, which are modeled by logical zero and one [4].

Therefore, to model Aristotelian logic with its "I don't know" or "I'm not sure" leads not only to the necessity of the introduction of the "third state", but also to a transition from digital to analog perception or from the point of view in terms of mathematics to the general idea of a more multiple or more multidimensional world.

For example, the state "I don't know" can be assessed with more or less depth—"how"much I don't know", "I know little or a lot about something".

Make an effort to approximate axiomatic positions suitable for computer modeling, using reproducing the basic functions of Boolean algebra "and" and "or" [5]. While in the basic logical functions we operate with arguments 0 and 1, for the functions of the ternary logic it is necessary to enter the state "U"—"Unknown".

2. The logic of Stephen Kleene

The mathematician Stephen Cole Kleene offered his example of the ternary logic with the state U [3].

At the same time he made a fundamental assumption, that the negation of U is also equal to U, NOT(U) = U.

In this case we will get functions with 9 possible states, for example, for the logical "and" and "or" functions (**Tables 1** and **2**).

Table 1. Possible states for the logical "and" function.

Status number	First argument	Second argument	Result
0	0	0	0
1	0	1	0
3	1	0	0
4	1	1	1
5	0	U	0
6	U	0	0
7	1	U	U
8	U	1	U
9	U	U	U

Table 2. Possible states for the logical "or" function.

Status number	First argument	Second argument	Result
1	0	0	0
2	0	1	1
3	1	0	1
4	1	1	1
5	0	U	U
6	U	0	U
7	1	U	1
8	U	1	1
9	U	U	U

But the task to introduce in such a scheme the concept of negation of U, i.e., NOT(U), is much more complex.

As we have already noticed, in the logic of Kleene, NOT(U) = U. But in general this is not a completely correct axiomatic assumption. For example, it is possible to create a connection between the digital and the analog world only if U is a subset of the set Y, where NOT(U) is the complement of the set U to the set Y. From the point of view of the probability measure, it is logical to assume that p(NOT(U)) = 1 − p(U). That is, the sum of sets U and NOT(U) completely forms the Y.

For example, the function "and" is given by the following probability polynomial.

$$P(AND, x1,x2) = P(x1)P(x2),$$

where P(AND) is the probability of a logical unit appearing for function AND, P(x1) is the probability of a logical unit appearing for the first argument, P(x2) is the probability of a logical unit appearing for the second argument. Function OR

$$P(OR, x1, x2) = P(x1) + P(x2) - P(x1)P(x2).$$

Let's arbitrarily set, that the probability p(u) equal to 0.25, p(U) = 0.25.

Table 3. Possible states for the logical "and" function, p(U) = 0.25, P(AND, x1, x2) = P(x1)P(x2).

Status number	First argument (x1)	Second argument (x2)	Result
0	0	0	0
1	0	1	0
3	1	0	0
4	1	1	1
5	0	0.25	0
6	0.25	0	0
7	1	0.25	0.25 = p(U)
8	0.25	1	0.25
9	0.25	0.25	0.0625 (UxU)

Table 4. Possible states for the logical "OR" function, P(OR, x1, x2) = P(x1) + P(x2) − P(x1)P(x2).

Status number	**First argument**	**Second argument**	**Result**
1	0	0	0
2	0	1	1
3	1	0	1
4	1	1	1
5	0	0.25	0.25 = p(U)
6	0.25	0	0.25
7	1	0.25	1
8	0.25	1	1
9	0.25	0.25	0.4375

In **Table 4**, we calculate the values of the polynomial for the OR function.

To align with the logic of Kleene, we will assume that probability values less than 0.5 correspond to the state of U. Thus, **Tables 3** and **4** correspond to **Tables 1** and **2**. Now we will consider the important function of "excluding or", denoting it XOR [4,6].

$$\text{XOR}(x, y) = (\text{NOT}(x)\&y)\ (\text{OR})(x\&\text{NOT}(y)) \tag{1}$$

We recall that "exclusive or" provides a universal tool for implementing encryption algorithms and other algorithms for protecting and converting information: in case we use the "exclusive or" function to "mix" a certain digital sequence with the initial one, in order to perform a reverse decryption operation we need to add the same sequence to the converted sequence.

$$Y = A(\text{XOR})B, \text{ then } A = Y(\text{XOR})B.$$

In **Table 5** we will adjust the Kleene's logic, taking into account the fact, that XOR (1, 1) = 0. We will determine, that the ninth state of XOR (U, U) = 0.

Table 5. Possible states for the "exclusive or" function (Use state NOT(U).

Status number	**First argument**	**Second argument**	**Result**
1	0	0	0
2	0	1	1
3	1	0	1
4	1	1	0
5	0	U	U
6	U	0	U
7	1	U	NOT(U)
8	U	1	NOT(U)
9	U	U	0

The Possible states for the XOR(x, 1)=NOT(x)&1!x&0=NOT(x), but in the logic of the Kleene it will turn out U.

Therefore, we cannot use Kleene's logic to create unambiguous transformations, designed among other means for the encryption and for the other necessary information protection algorithms. Thus, we must automatically either "switch to" the tetrarch (four-digit) logic, or conditionally "move" into the space of a continuous measure where the value of U and NOT (U) belongs to the interval [0,1].

Now is possible to formulate the fundamental statement inherent to any multidimensional logic.

3. Regarding the construction of any multidimantional logic

When constructing logics with more than two dimensions in order to implement bijective (one-to-one) mappings, it is necessary to include the additional negation state, which increases the multidimensionality of the logic or allows the transition to the space of conditionally continuous states. It is interesting to consider the transfer of Boolean functions to probabilistic ones, which would make possible the description of different situations using continuous polynomials over the field of real numbers, including those related to the probabilistic distribution of various different states, including those comprising U.

Function NOT

$$P(NOT, x) = 1 - P(x).$$

Denote

$$P(x1) = P1 \text{ and } P(x2) = P2.$$

According to the Equation (1)

$$p(XOR, p1, p2) = ((1 - p1)p2) + (p1(1 - p2) - ((1 - p1)p2)(p1(1 - p2)).$$

Then

$$P(XOR, x1, x2) = P1 + P2 - 3P1P2 + P1^2P2 + P1P2^2 - (P1P2)^2.$$

Based on the obtained formula, it is interesting to remark notice that "exclusive or" has the property of statistical alignment—if any of the arguments is 0.5, i.e., the probability of "one" and "zero" are equal, then the probability of "one" appearing "at the output" does not depend on the second argument.

Thus, by setting, for example, U = 0.25, we get

$$P(NOT, 0.25) = 0.75,$$

$$P(AND,1, 0.25) = 0.25,$$

$$P(OR, 1, 0.25) = 1 + 0.25 - 0.25 - 1,$$

$$P(XOR, 1, 0.25) = 1 + 0.25 - 3 \times 0.25 + 12 \times 0.25 + 1 \times (0.25)^2 - (1 \times 0.25)^2 = 0.75,$$

which corresponds to **Table 5** (NOT(U)) and corresponds to the proposed logic.

$$P(XOR,0.25, 0,25) = 0.28125.$$

For the further calculations we will consider this value to be 0.

For the calculations within the framework of the ternary logic, it is possible to use advanced floating-point computing tools [5,7] and configure the states number 9 in the tables of functions in accordance with the described logic.

4. Conclusion

The use of the ternary logic requires the correct implementation of the state NOT(U), which automatically puts the calculator into the tetrarch state.

The development of multidimensional logics becomes possible when using calculations in real numbers, including the transformation of Boolean functions into probabilistic polynomials.

So, in this article the authors have analyzed the addition of the “third state” to the binary logic, and for that purpose they have chosen the approach of Stephen Kleene, who had introduced the “indefinite state”. Further it was shown that it was necessary to introduce in the main points of Kleent’s concept also the negation of the “indefinite state”. So the authors have proposed and formulated the method of the substitution of logical functions by probabilistic polynomials based on the field of real numbers that are conveniently calculated on modern hardware, for example, in video card processors.

Calculations in ternary logic can be useful for the implementation of new artificial intelligence projects that model the operation of thinking, mental action, or the workings of the human mind with uncertain results, while the transition to probabilistic functions can expand the capabilities of models and simplify the analysis of errors that occur during the operation of artificial intelligence systems.

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