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Article

Decision-making models under conditions of uncertainty of formation, description and intellectual analysis of complex data files

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Abstract: Research objective: to prove the feasibility of forming a problem-oriented array under complex conditions of uncertainty by using different options for modeling decision-making and selecting the optimal model. Formation, description, and intellectual analysis of a complex data set, which is an example of a problem-oriented library-museum-archival-information array on nobelistics, are carried out under conditions of uncertainty due to the ambiguity of attribution of each element to this array. The possibility of modeling decision-making in these conditions is shown, the best of which is the optimal formation, description, and intellectual analysis of a complex array of problem-oriented data. A typical information situation is used for modeling when the decision-making body has knowledge of the a priori probability distribution on the state elements of the data array. For each of the seven variants of information situations, a set of criteria for making optimal decisions is selected; each criterion is mathematically described. The real functioning subject-oriented library-museum-archive-information data array on nobelistics of the International Nobel Information Center, consisting of the Nobel Scientific Library, the Museum of the Nobel Family and Nobel Prize Laureates, the Archive of the Nobel Family and Nobel Prize Laureates, and electronic databases on nobelistics, was used.

Keywords: formation; description and intellectual analysis of data; library-museum-archival-information array on nobelistics; uncertainty conditions; decision-making; models

1. Introduction

The peculiarity of decision-making processes is to take into account the presence of a person, a collective of persons, or a decision-making body that seeks to achieve some goal on the basis of their preferences about values and with the help of automated decision support systems. In the theory of decision-making, the most preferable solution is considered to be the one that is consistent with the structure of preferences of the decision-making body, as well as with the information it has about the decision-making problem. In this case, the theory makes it possible to build normative procedures that help the decision-making body to formalize its preferences, and decision-making is reduced to a comparison of those properties of the solutions that are the basis for evaluating [1–3]. Several models of decision-making are well known: the rational model, the intuitive model, the Vroom-Yetton model, the bounded rationality model, the game theory model, the neurobiological model, and others [4–6].

The quality of the decision-making process is in direct dependence on the completeness of taking into account all the factors that are essential for the consequences of the decisions made. Often these factors are subjective in nature, inherent in both the decision maker and any decision-making process. Hence the conditions of uncertainty in decision-making: the decision-making body has less information than is necessary for the expedient organization of its actions in the decision-making process. Directly with the conditions of uncertainty we face when it is necessary to carry out the formation, description, and intellectual analysis of data representing a complex problem-oriented library-museum-archive-information array (LMAIA) on nobelistics, functioning in real time [7–13]. Uncertainty arises due to the ambiguity of attributing each element of nobelistics to this array.

Our experience allows us to propose the following classification of uncertainties:

1) essence uncertainty (lies in the essence of the studied objects and/or processes);
2) uncertainty generated by the total number of objects (elements, processes) included in the situation under study;
3) uncertainty caused by the lack of information and data on its reliability due to technical, social, or other reasons;
4) uncertainty caused by too high or inaccessible payment for certainty;
5) uncertainty generated by the decision-making body due to its lack of experience and knowledge of factors affecting decision-making;
6) uncertainty related to limitations in the decision-making situation (limitations on time and space elements of parameters characterizing decision-making factors);
7) uncertainty caused by the behavior of the environment influencing the decision-making process.

Thus, in decision-making processes there are a number of situations that have a certain degree of uncertainty and require for their description such a mathematical apparatus, which would a priori include the possibility of formalizing uncertainties and would allow performing the actions necessary to achieve the goal [10–13].

Historically, the first was the apparatus of probability theory, according to which the uncertainty of a situation is described by some normalized measure characterizing the possibility of occurrence of predetermined random outcomes (elements or subsets of some set).

A natural continuation of probabilistic methods for describing uncertain situations was game theory [14–17], in which uncertainty was generated by conflict and opposing interests of players bound by the rules of the game, and statistical decision theory [17], in which a passive environment or “nature” was chosen as one of the players, whose behavior was characterized by given laws of probability distribution. These theories are extreme cases of different degrees of uncertainty gradation or information situations.

Another class of uncertain situations is based on the concept of a vague (fuzzy) set introduced by Zadeh [18]. This apparatus is adequate for describing such situations that do not have strictly defined boundaries, so it is used for the work of artificial intelligence. Schemes for constructing a general mathematical apparatus describing a wide class of uncertain situations are given in [19–21]. A widely known and widespread model is the static model of decision-making based on the game-theoretic concept [22], applicable in many real situations of ad hoc selection of options, plans, tuples, actions, alternatives, strategies, etc., associated with the uncertain influence of the environment on the situation of their selection by the decision-making body.

2. Library-museum-archival-information array on nobelistics

Data in the local LMAIA on nobelistics, belonging to the International Nobel Information Center (INIC) [23], are presented in the following quantity: about 10 thousand books and brochures in the Nobel Scientific Library, more than 6 thousand exhibits in the Museum of the Nobel Family and Nobel Prize Laureates, more than 100 thousand sheets of documents in the Archive of the Nobel Family and Nobel Prize Laureates, and 20 databases of electronic documents on nobelistics with a volume of 2 terabytes. This is more than enough to form an information problem-oriented system with a high level of accuracy and completeness of search in all areas of nobelistics.

To form precise queries on nobelistics, a special thesaurus was initially developed, the terms of which are easily converted into keywords entered in a search query. Other keywords are not perceived by the system. In the description of each element of the system (book, brochure, museum exhibit, archival document, etc.), a specially developed classification technology is used, the main parameter of which is the surname (and names) of each of thousands of Nobel Prize winners. Any new element is placed in the system according to this parameter; it must necessarily be associated with at least one surname of a Nobel laureate.

3. Initial conditions of modeling

When studying static models of decision-making under uncertainty, we proceed from the scheme assuming the presence of: 1) the control body U has a set of mutually exclusive decisions $\Phi = \{\varphi_1,\ldots,\varphi_m\}$, one of which it needs to make; 2) the environment C has a set of mutually exclusive states $\Theta = \{\theta_1,\ldots,\theta_n\}$, but in which particular state the environment C is (or will be) the control body U does not know; 3) the control body U is evaluated by a functional $F = \{f_{jk}\}$ characterizing its “gain” or “loss” when choosing a decision $\varphi_k \in \Phi$ if the environment C is (or will be) in the state $\theta_j \in \Theta$. In our case, the environment C is a complex array of data LMAIA, and we consider the optimal formation, description, and intelligent analysis of data as a gain.

Under this scheme, the quantitative side of the theory of decision-making in conditions where the environment “behaves” in an antagonistic way with respect to the choice of decisions by the control body U (state of uncertainty) is usually called game theory [22]. In the case of “passive” environment (“passive nature”), about which the control body U knows the probability distribution $p = \{p_1,\ldots,p_n\}$ on $\Theta = \{\theta_1,\ldots,\theta_n\}$, it is accepted to call games with nature or static decisions. These cases of environmental behavior can be called extreme cases. In the general case, there is a significant gradation of situations that determine the strategy of behavior of the environment C.

The definition and classification of these situations form the basis of the theory of decision-making under uncertainty, since they partially allow us to solve the well-known problem of choosing a decision-making criterion by developing for each situation a set of such criteria.

Our approach to the process of decision-making by the control body U consists of 1) forming a set of decisions Φ and a set of states of the environment Θ; 2) defining and setting the main efficiency and utility indicators included in the calculation of the evaluation functional $F = \{f_{jk}\}$; 3) defining by the control body U the situation characterizing the strategy of behavior of the environment *C*; 4) choosing a decision-making criterion from the set of criteria characterizing the situation defined by the control body *U*; 5) making an optimal decision according to the chosen criterion or correcting it (if the optimum decision is optimal); 6) choosing a decision-making criterion from the set of criteria characterizing the situation defined by the control body *U*; 7) making an optimal decision according to the chosen criterion or correcting it (if the *optimum* decision is optimal).

The formal component of the decision-making process under conditions of uncertainty consists of the production of calculations of performance indicators included in the definition of the evaluation functional $F = \{f_{jk}\}$ and in the production of calculations to find the optimal solution $\varphi^0 \in \Phi$ (or a set of such solutions $\overline{\Phi} \in \Phi$) according to a given decision-making criterion. Algorithms for the calculation of efficiency indicators and decision-making criteria with the use of modern computer systems constitute the *mathematical support of* the static process of decision-making under conditions of uncertainty. Algorithms of formation on the basis of application of information means and modern computer systems of the information picture in the control body *U*, characterizing the strategy of behavior of the environment *C*, and providing the definition of the situation constitute the *information support of* the static process of decision-making in conditions of uncertainty.

Let us define the basic elements of static models of decision-making processes.

Under the situation of decision-making we understand $\{\Phi, \Theta, F\}$, where $\Phi = \{\varphi_1,\ldots,\varphi_m\}$ is the set of decisions of the control body *U*; $\Theta = \{\theta_1,\ldots,\theta_n\}$ is the set of states of the environment *C*, which can be in one of the states $\theta_j \in \Theta$; and $F = \{f_{jk}\}$ is the evaluation functional (matrix of the evaluation functional) defined on $\Theta \times \Phi$ and taking values from R^1, at that $f_{jk} = f(\theta_j, \varphi_k)$. In the extended form, the situation of decision-making is characterized by a matrix, the elements of which f_{jk} are quantitative evaluations of the taken decision $\varphi_k \in \Phi$ under the condition that the environment *C* is in the state $\theta_k \in \Theta$:

$$\begin{array}{cccccc} & \varphi_1 & \cdots & \varphi_k & \cdots & \varphi_m \\ \theta_1 & f_{11} & \cdots & f_{1\mathrm{k}} & \cdots & f_{1\mathrm{m}} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \theta_j & f_{j1} & \cdots & f_{jk} & \cdots & f_{jm} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \theta_n & f_{n1} & \cdots & f_{nk} & \cdots & f_{nm} \end{array}$$

Such concepts as efficiency, utility, losses, risk, etc., are closely related to the category of estimated function. The choice of the form of expression of the evaluative function depends on the purpose and objectives of the management of the object *O*, the availability of methods for obtaining and calculating the effectiveness of tasks solved by the management object *O* and the management body *U*, the time of the process of preparation and decision-making, etc. Most often, two forms of expression of the evaluation function are used: *F*, defining utility, value, etc., or losses, damages, risk, etc. The evaluative function *F* has a positive ingredient if the decision-making body *U* proceeds from the condition of achieving $\max\limits_{\varphi_k \in \Phi}\{f_{jk}\}$. In this case, for the positive ingredient, we will use the notation $F = F^+\{f_{jk}^+\}$. For a negative ingredient *F*, the governing body *U* assumes the condition of achieving $\underset{\varphi_k \in \Phi}{min}\{f_{jk}\}$ when making the decision. In this case, $F = F^{-}\{f_{jk}^{-}\}$.

The identification of positive and negative ingredients is characteristic of actively directed systems. These are the systems that provide the solution to the problems of attributing each element of nobelistics to LMAIA. We can note a number of interesting situations in which, for example, the ingredient of a passively directed system can be determined from the condition of reaching $\lambda \min_{\varphi_k \in \Phi}\{f_{jk}\} + (1-\lambda)\max_{\varphi_k \in \Phi}\{f_{jk}\}(0 \leq \lambda \leq 1)$, and at $\lambda = 0$ we have $F = F^{+}$, and at $\lambda = 1$ we have $F = F^{-}$.

The definition of the evaluative function in the form of F^{+}, is usually used to express the categories of utility, gain, efficiency, probabilities of achieving target events, etc.; in contrast, F^{-} is used to express loss, regret, damage, risk, etc. Note that when forming the evaluation functional, the expression of the ingredient is determined by the management and decision-making purpose of the *U* body. It is clear that the positive form of expression of the ingredient of the evaluation function is more often used (F^{+}). However, in some cases a negative value is necessary.

The regret function is a linear transformation of a positive or negative value of an ingredient of the evaluation functional to relative units. Such transformation sets the origin of the evaluation functional "zero" for each state of the environment θ_j: 1) for F^{+}, the case of a fixed state of the environment $\theta_j \in \Theta$, the value $l_j = \max_{\varphi_k \in \Phi} f_{jk}^{+}$ is found, and the regret function is defined in the form $r_j(\phi_k) = l_j - f_{jk}^{+}$; 2) for F^{-}, the case of a fixed state of the environment $\theta_j \in \Theta$, the value $L_j = \min_{\varphi_k \in \Phi} f_{jk}^{-}$ is found, and the regret function is defined in the form $r_j(\phi_k) = f_{jk}^{-} - L_j$.

The regret function has a negative form of the evaluation functional F^{-}, $r_j(\phi_k) \geq 0$, and $r_j = 0$ for at least one solution ϕ_k at $\forall\theta_j \in \Theta$.

4. Information situations and decision-making criteria

Let us introduce decision-making situations, which are formalized by the model in the form of a tuple $\{\Phi, \Theta, F\}$, which makes it possible to define various information situations. By an *information situation I* we understand a certain degree of uncertainty in the choice of the environment *C* of its states from a given set Θ, which is available to the control body *U* at the time of decision-making. Let us define a classifier of information situations that characterize the "behavior" of the environment *C* in the decision-making process when choosing its states $\theta_j \in \Theta$.

Let's introduce *7 information situations*:

I_1 is the first information situation characterized by a given distribution of a priori probabilities on the elements of the set Θ;

I_2 is the second information situation characterized by a given probability distribution with unknown parameters;

I_3 is the third information situation characterized by the given systems of linear relations of orders on the components of the a priori distribution of states of the environment *C*;

I_4 is the fourth information situation characterized by an unknown probability distribution on the elements of the set Θ;

I_5 is the fifth information situation characterized by antagonistic interests of the environment C in the decision-making process;

I_6 is the sixth information situation characterized by "intermediate" between I_1 and I_5 cases of the environment's choice of its states;

I_7 is the seventh information situation characterized by a fuzzy set of states of the environment C.

These situations are generalized characteristics of the uncertainty levels of the states of the environment C. Different gradations of uncertainty in each information situation are used in the study of decision criteria.

Under the *decision criterion* $\chi \in K$, we will understand an algorithm that determines for each decision-making situation $\{\Phi, \Theta, F\}$ and information situation I the only optimal solution $\varphi^0 \in \Phi$ or a set of such solutions $\overline{\Phi} \subset \Phi$, which we will call equivalent according to the given decision criterion. The decision criterion can be considered as a preference operation on the set of solutions Φ, taking into account the element of uncertainty of the possible states $\theta_j \in \Theta$ of the environment C, ordering the set of solutions Φ into a transitive sequence in the order of preference.

Thus, any information situation I is characterized by a set of decision criteria $K_{I_i} = \{\chi_{si}\}$ (I = 1,…,7). For example, for the first information situation, the composite criteria are Bayesian, maximum likelihood, modal, minimum variance, etc. (**Table 1**).

Table 1. Correspondence of decision-making criteria to information situations.

No	Characterization information situation	Decision-making criteria
1	A distribution of a priori probabilities on the elements of the set Θ is given	1. Bayes criterion 2. Maximum likelihood criterion 3. modal criterion 4. Minimum variance criterion 5. Criterion of minimum entropy of mathematical expectation 6. Modified criterion
2	A probability distribution with unknown parameters is given	1. Parametric Bayes criterion 2. Parametric criterion of maximum likelihood of the estimated functional (EF) 3. Parametric criterion of minimum variance of the EF 4. Parametric modal criterion 5. Parametric criterion of maximum entropy of the mathematical expectation of the EF
3	A system of linear order relations on the components of the a priori distribution of the state of the medium is specified	Determines the type of order relationship, set by the decision-making body U based on the information at its disposal, its experience, the situation, and the conditions of the decision-making environment
4	We do not know the probability distribution on the elements of the set Θ	1. Criterion for maximal measures of Bayesian sets 2. Maximum of the integral Bayesian value of the EF 3. largest integral potential 4. Bernoulli-Laplace criterion 5. Khomenyuk criterion 6. Gibbs-Janes criterion

Table 1. (*Continued*).

No	Characterization information situation	Decision-making criteria
5	Antagonistic interests of environment *C* in the decision-making process	1. Wald criterion 2. Savage's criterion 3. Uncertainty function
6	Intermediate cases of environment *C's* choice of *its* states	1. Hurwitz criterion 2. Hodges-Lehmann criterion 3. Menges criterion 4. Schneeweiss criterion
7	Fuzzy set of environment states	1. Bringing the subjective probability distribution of the values of the components of the belonging function components 2. Criterion of the type of probability distribution type of the EF values 3. Criterion of type of dispersion of EF values 4. Modal type criterion

In a given situation $\{\Phi, \Theta, F\}$, the decision-making problem is that the decision authority *U must* choose one decision that is optimal according to the chosen criterion. The axiomatic decision-making problem is characterized mainly by three factors: $\{I, K_I, A\}$, where I is the information situation, K_I is the set of decision criteria corresponding to the information situation I, and A is the system of axioms for analyzing decision criteria. The axiomatic approach in the analysis of decision-making criteria is understood as a method of selecting the most acceptable axioms (postulates), which allow the management body U to investigate the problems of decision-making in the uncertainty of finding a suitable decision-making criterion. Decision-making in this situation $\{\Phi, \Theta, F\}$ is largely facilitated by the possibility of determining the information situation I, as well as the establishment of a system of axioms for selecting the criterion $K_{I_i} = \{\chi_{si}\}$ (i = 1,…,7).

To date, axiom systems do not exist for all information situations, and the choice of criterion in a given information situation I based on the existing axiom system can be ambiguous. The ambiguity of criterion selection is determined by the incompleteness of the axiom system. Despite the presence of these features that hinder the resolution of the decision-making problem, we note that each of the information situations under consideration is characterized by a "potential" criterion that reflects the main tendencies of decision-making in this or that information situation.

The main tendency of researching the problem of decision-making consists in detailing and classifying information situations, on the one hand, and in developing criteria for these information situations with some elements of researching their positive and negative sides in the issues of efficiency of functioning of the management object O and the management body U.

5. Models for the first information situation

The information situation I_1 characterizes the case when the decision-making body U has knowledge of the a priori probability distribution $p=\{p_1,\ldots,p_n\}$, $p_j = P\{\theta$

$= \theta_j\}$, $\sum_{j=1}^{n} p_j = 1$ on the elements $\theta_j \in \Theta$ of the states of the environment C. This situation is the most common information situation identifying the "behavior" of the environment C in most practical decision-making tasks under "risk" conditions. Its introduction into the decision-making processes allowed for effectively using constructive methods of probability theory in the development of statistical decision theory.

In practical problems, the calculation of a priori distribution of medium states C *is* carried out either by processing extensive statistical material or by analytical methods based on the formulation of hypotheses of medium behavior with the subsequent use of basic axioms, theorems, and methods of probability theory. Both ways are approximate, because in practice, due to a number of limitations (in terms of cost, expenses, time, and space), there are difficulties in obtaining and processing statistical material, the formulated system of hypotheses of the behavior of the medium is inherently incomplete, and when using "working" hypotheses, it is necessary to make appropriate assumptions (for example, about the independence of events) to the detriment of the physics of the process in order to carry out the calculation p. Among the various concepts of probability, such an a priori distribution p is commonly referred to as an *objective probability*.

However, in a number of statistical decision-making processes, due to the complexity of the "behavior" of the environment C, the lack of collection and processing of statistical material, the use of analytical methods, etc., the decision-making body U, relying on its experience or on the opinion of a group of experts, prefers to use the concept of probability p, developed on the basis of the idea of the degree of certainty about a given factor, feature, or symptom characterizing the properties of the "behavior" of the environment. This definition of a priori distribution p, which made the concept of probability a matter of opinion, was called *subjective probability*. This is what happens every time a staff member introduces a new element into one of the LMAIA structures.

On the basis of taking into account possible errors and inaccuracies, as well as the ambiguity of opinions of the group of experts when calculating the a priori distribution, we synthesize optimal decisions on the a priori distribution $p = \{p_1,\ldots,p_n\}$, taking values from a flat set of $\Delta_n = \left\{p{:}0 \le p_j \le 1, \sum_{j=1}^{n} p_j = 1\right\}$.

Let us consider the basic criteria for decision-making in an information situation I_1 characterized by the probability distribution $p_j = P\{\theta = \theta_j\}$, $\sum_{j=1}^{n} p_j = 1$ of the states $\theta_j \in \Theta$ of the environment C. Let a decision situation $\{\Phi, \Theta, F\}$ be given in which the evaluation functional $F = \{f_{jk}\}$ belongs to the class F^- or F^+, and sets Φ and Θ are given in the form of $\Phi = \{\varphi_1,\ldots,\varphi_m\}$, and $\Theta = \{\theta_1,\ldots,\theta_n\}$.

1) *Bayes criterion*. The essence of this criterion is to maximize the mathematical expectation of the estimated functional, transforming the formulas of a priori probabilities into a posteriori ones. Optimal solutions $\varphi_{ko} \in \Phi$ (or a set of such optimal solutions) are those solutions for which the mathematical expectation of the estimated functional reaches the largest possible value:

$$B^{+}(p,\varphi_k)=\max_{\varphi_k\in\Phi}B^{+}(p,\varphi_k)=\max_{\varphi_k\in\Phi}\left[\sum_{j=1}^{n}p_j\,f_{jk}^{+}\right]=\sum_{j=1}^{n}p_j\,f_{jko}^{+}.$$

If the maximum is achieved on several solutions of Φ, the set of which we denote by $\overline{\Phi}$, then such solutions will be called equivalent.

The value $B^{+}(p,\varphi_k)=\sum_{j=1}^{n}p_j f_{jk}^{+}$ is called the value of the Bayes estimator for the solution φ_k. The great popularity of this criterion in the information situation I_1 is explained by the fact that the Bayes criterion is closely related to the axioms of utility theory (axiom of Nyman and Morgenstern), in which the total utility is defined as the mathematical expectation of private utilities. If the valuation functional is given in the form of F^{-}, then *min* is used instead of the operation *max* of the mathematical expectation. If the valuation functional is given in regret or risk, the corresponding value $B^{-}(p,\varphi_k)$ is usually called the Bayesian risk for the solution $\varphi_k\in\Phi$.

2) *Criterion for maximizing the probability of distribution of the estimated functional*. Fix the value α, satisfying the inequalities $\alpha_1<\alpha<\alpha_2$, where $\alpha_1=\min_k\min_k f_{jk}^{+}$, $\alpha_2=\max_k\max_k f_{jk}^{+}$, ($j$ = 1,...,n; k = 1,...,m).

For each solution $\varphi_k\in\Phi$, let us determine the probability $P\left(f_{jk}^{+}\geq\alpha\right)$ that the value of the evaluation functional is not less than α for the state of the environment $\theta_j\in\Theta$ and the solution $\varphi_k\in\Phi$. The essence of this criterion is to find a solution $\varphi_{ko}\in\Phi$ (or a set of such solutions $\overline{\Phi}$) for which $P\left(f_{jk}^{+}\geq\alpha\right)=\max_{\varphi_k\in\Phi}P\left(f_{jk}^{+}\geq\alpha\right)$. When using this criterion, the control body U proceeds from a specific value of α and considers as optimal those solutions $\varphi_{ko}\in\Phi$ for which this condition is met.

For fixed α and φ_k, the inequality $f_{jk}^{+}\geq\alpha$ defines the set of states of the environment $\Theta_{\alpha,k}$. Then the probability $P\left(f_{jk}^{+}\geq\alpha\right)$ is

$$P\left(f_{jk}^{+}\geq\alpha\right)=P\left(\theta\in\Theta_{\alpha,k}\right)=\sum_{\theta_j\in\Theta_{\alpha,k}}p(\theta=\theta_j).$$

In this criterion, the magnitude α is given by the control U. Therefore, the set $\overline{\Phi}$ depends on α, i.e., $\overline{\Phi}=\overline{\Phi}(\alpha)$. For two values of α^{*} and α^{**} such that $\alpha_1\leq\alpha^{*}\leq\alpha_2$, $\alpha_1\leq\alpha^{**}\leq\alpha_2$, and $\alpha^{*}\leq\alpha^{**}$, we have $\overline{\Phi}(\alpha^{**})\subseteq\overline{\Phi}(\alpha^{*})$. Furthermore, $P\left(f_{jk}^{+}\geq\alpha^{*}\right)\geq P\left(f_{jk}^{+}\geq\alpha^{**}\right)$.

If the evaluation functional is given in the form of $F=F^{-}$, then for each decision $\varphi_k\in\Phi$, the probability of $P\left(f_{jk}^{-}\leq\beta\right)$ is defined, and the application of the criterion consists in selecting decisions φ_{ko} or $\overline{\Phi}(\beta)$, for which $P\left(f_{jk}^{-}\leq\beta\right)=\max_{\varphi_k\in\Phi}P\left(f_{jk}^{-}\leq\beta\right)$, where the value of β, such that $\alpha_1\leq\beta\leq\alpha_2$, is given by the decision authority U.

3) *Criterion of minimum variance of the estimated functional*. For each solution $\varphi_k\in\Phi$, we define the mean value $B^{+}(p,\varphi_k)$ of the evaluation functional and the variance σ_k^2 in the form of $B^{+}(p,\varphi_k)=\sum_{j=1}^{n}p_j f_{jk}^{+}$, namely

$$\sigma_k^2 = \sigma^2(p,\varphi_k) = \sum_{j=1}^{n}\left[f_{jk}^{+} - B^{+}(p,\varphi_k)\right]^2 p_j.$$

Dispersion σ_k^2 characterizes the dispersion of a random variable of the value of the estimated functional for the solution φ_k with respect to the mean value $B^{+}(p,\varphi_k)$. The essence of this criterion is to find a solution φ_{ko} (or a set of solutions Φ) for which $\sigma^2(p,\varphi_k) = \min_{\varphi_k\in\Phi}\sigma^2(p,\varphi_k)$.

The main disadvantage of this criterion is that the variance at the solution $\varphi_{k1} \in \Phi$ may be smaller than at the solution $\varphi_{k2} \in \Phi$, i.e., $\sigma_{k1}^2 \le \sigma_{k2}^2$, while $B^{+}(p,\varphi_{k1}) < B^{+}(p,\varphi_{k2})$. This suggests that the criterion of minimum variance of the estimated functional, on the one hand, is in a sense an auxiliary criterion, and on the other hand, if it is accepted, it is necessary to further define this criterion by slightly modifying the form of σ_k^2, for example, in one of the following ways:

$$\sigma_{(p,\varphi_k)}^2 = \sum_{j=1}^{n}\left[f_{jk}^{+} - \max_{\varphi_s\in\Phi}B^{+}(p,\varphi_s)\right]^2 p_j,$$

$$\sigma_{(p,\varphi_k)}^2 = \sum_{j=1}^{n}\left[f_{jk}^{+} - \frac{1}{m}\sum_{s=1}^{m}B^{+}(p,\varphi_s)\right]^2 p_j.$$

If the evaluation functional is given in the form $F = F^{-}$, then the solution φ_{ko} to the minimum of the evaluation functional is found from the condition $\sigma^2(p,\varphi_{ko}) = \min_{\varphi_k\in\Phi}\sigma^2(p,\varphi_k)$. where the value $\sigma^2(p,\varphi_{ko})$ is determined in one of the following ways: $\sigma^2(p,\varphi_k) = \sum_{j=1}^{n}\left[f_{jk}^{-} - B^{-}(p,\varphi_k)\right]^2 p_j$, $\sigma^2(p,\varphi_k) = \sum_{j=1}^{n}\left[f_{jk}^{-} - \min_{\varphi_s\in\Phi}B^{-}(p,\varphi_s)\right]^2 p_j$, $\sigma^2(p,\varphi_k) = \sum_{j=1}^{n}\left[f_{jk}^{-} - \frac{1}{m}\sum_{s=1}^{m}B^{-}(p,\varphi_s)\right]^2 p_j$.

4) *Modal criterion*. The essence of this criterion is that the control body U proceeds from the most probable state of the environment. Suppose that there is a single value of $p_{j_1} = \max_{\theta_j\in\Theta}P(\theta = \theta_j)$.

Using this criterion, the control body U assumes that the environment is in the state $\theta_{j_1} \in \Theta$, and the optimal φ_{ko} or $\overline{\Phi}$ is determined from the condition $f_{j_1k_0}^{+} = \max_{\varphi_k\in\Phi}f_{j_1k}^{+}$. If it turns out that the maximum $P(\theta = \theta_j)$ is achieved at a priori probabilities $p_{j_1}, p_{j_2},..., p_{j_s}$, then the optimal solution φ_{ko} (or $\overline{\Phi}$) is determined from the condition $\frac{1}{s}\sum_{\gamma=1}^{s}f_{j_\gamma k_0} = \max_{\varphi_k\in\Phi}\frac{1}{s}\sum_{\gamma=1}^{s}f_{j_\gamma k}^{+}$.

The main drawback of this criterion is the possibility that if we take two solutions φ_{k1} and φ_{k2}, for which $f_{j_1k_1}^{+}f_{j_1k_2}^{+}$, then according to this criterion the solution φ_{k1}, i.e., $\varphi_{k1} > \varphi_{k2}$, is preferred. However, it may turn out that $B^{+}(p,\varphi_{k1}) < B^{+}(p,\varphi_{k2})$.

The main advantages of this criterion are 1) sufficiency of identifying only the most probable states of the environment, and it is not necessary to know the quantitative values of the probabilities of realization of these states, and 2) determination (calculation) of the estimated functional only for the most probable states of the environment, which increases the speed of decision-making many times. It should be noted that when setting the estimated functional F in the form F^{-}, the operation *max* is replaced by *min*.

5) *Criterion of minimum entropy of the mathematical expectation of the estimated functional*. Suppose that $f_{jk}^{+} > 0$ for all j and k. Let us define the entropy of the mathematical expectation of the estimated functional for the solution $\varphi_k \in \Phi$ as follows:

$$\mathrm{H}(p, \varphi_k) = -\sum_{j=1}^{n} \left(\frac{p_j f_{jk}^{+}}{\sum_{j=1}^{n} p_j f_{jk}^{+}} \right) \ln \left(\frac{p_j f_{jk}^{+}}{\sum_{j=1}^{n} p_j f_{jk}^{+}} \right).$$

The essence of this criterion consists in finding a solution φ_{ko} (or $\overline{\Phi}$) from the condition $\mathrm{H}(p, \varphi_{ko}) = \min_{\varphi_k \in \Phi} \mathrm{H}(p, \varphi_k)$. In case of non-fulfillment of the condition $f_{jk}^{+} > 0$ for all j and k, the transition from the values f_{jk}^{+} of the estimated functional to the risk (regrets, losses) of the form $\breve{f}_{jk}^{-} = \max_{\substack{\theta_j \in \Theta \\ \varphi_k \in \Phi}} f_{jk}^{+} - f_{jk}^{-}$, is made, and the solution φ_{ko} is found from the condition $\varphi_k \in \Phi$ of entropy minimum of the mathematical expectation of the estimated functional of the form $H(p, \phi_{ko})$ at $\breve{f}_{jk}^{-} > 0$:

$$\mathrm{H}(p, \varphi_k) = -\sum_{j=1}^{n} \left(\frac{p_j \breve{f}_{jk}^{-}}{\sum_{j=1}^{n} p_j \breve{f}_{jk}^{-}} \right) \ln \left(\frac{p_j \breve{f}_{jk}^{-}}{\sum_{j=1}^{n} p_j \breve{f}_{jk}^{-}} \right).$$

6) *Modified criterion*. We fix the value of λ, satisfying the condition $0 \leq \lambda \leq 1$. For each of them $\varphi_k \in \Phi$, we define the value of $\chi(p, \varphi_k) = (1 - \lambda)[B^{+}(p, \varphi_k)]^2 - \lambda \sigma^2(p, \varphi_k)$, where denotes $B^{+}(p, \varphi_k) = \sum_{j=1}^{n} p_j f_{jk}^{+}$, $\sigma^2(p, \varphi_k) = \sum_{j=1}^{n} \left[f_{jk}^{+} - B^{+}(p, \varphi_k) \right]^2 p_j$.

The essence of the modified criterion is to find a solution φ_{ko} (or a set of solutions $\overline{\Phi}$) from the condition $\chi(p, \varphi_{ko}) = \max_{\varphi_k \in \Phi} \chi(p, \varphi_k)$.

Note that in two special cases, $\lambda = 0$ and $\lambda = 1$. This criterion coincides with the Bayes criterion and with the criterion of minimum variance of the evaluation functional.

Let us introduce two quantities

$$\lambda^{*} = \min_{\varphi_k \in \Phi} \frac{\left[\sum_{j=1}^{n} p_j f_{jk}^{+} \right]^2}{\sum_{j=1}^{n} p_j \left(f_{jk}^{+} \right)^2}, \lambda^{**} = \max_{\varphi_k \in \Phi} \frac{\left[\sum_{j=1}^{n} p_j f_{jk}^{+} \right]^2}{\sum_{j=1}^{n} p_j \left(f_{jk}^{+} \right)^2}.$$

Obviously, the values of λ^{*}, and λ^{**} are such that the inequalities are satisfied $0 \leq \lambda^{*} \leq \lambda^{**} \leq 1$.

Lemma 1. *If a quantity* λ *satisfies the condition* $0 \leq \lambda \leq 1$, *then* $\chi(p, \varphi_{ko}) \geq 0$ *for any* $\varphi_k \in \Phi$.

The proof of this statement follows from the fact that

$$\chi(p, \varphi_k) = (1-\lambda)\left[\sum_{j=1}^{n} p_j f_{jk}^{+}\right]^2 - \lambda \sum_{j=1}^{n}\left[f_{jk}^{+} - \sum_{l=1}^{n} p_l f_{lk}^{+}\right]^2 p_j$$
$$= \left[\sum_{j=1}^{n} p_j f_{jk}^{+}\right]^2 - \lambda \sum_{j=1}^{n} p_j \left(f_{jk}^{+}\right)^2 \geq 0,$$

since $\lambda \leq \left[\sum_{j=1}^{n} p_j f_{jk}^{+}\right]^2 / \sum_{j=1}^{n} p_j \left(f_{jk}^{+}\right)^2$ for any solution $\varphi_k \in \Phi$ at $\lambda \in \left[0, \lambda^{*}\right]$.

As a corollary to the lemma, we obtain that $(1-\lambda)[B^{+}(p, \varphi_k)]^2 \geq \lambda \sigma^2(p, \varphi_k)$ at $\lambda \in \left[0, \lambda^{*}\right]$, i.e., at these values of λ, the modified criterion is more sensitive to the Bayesian criterion of maximizing the average payoff $B^{+}(p, \varphi_k)$ than to the criterion of minimizing the variance $\sigma^2(p, \varphi_k)$.

Lemma 2. *If a quantity* λ *satisfies the condition* $\lambda^{**} \leq \lambda \leq 1$, *then* $\chi(p, \varphi_{ko}) \leq 0$ *for any* $\varphi_k \in \Phi$.

The proof of this statement follows from the fact that

$$\chi(p, \varphi_k) = (1-\lambda)\left[\sum_{j=1}^{n} p_j f_{jk}^{+}\right]^2 - \lambda \sum_{j=1}^{n}\left[f_{jk}^{+} - \sum_{l=1}^{n} p_l f_{lk}^{+}\right]^2 p_j$$
$$= \left[\sum_{j=1}^{n} p_j f_{jk}^{+}\right]^2 - \lambda \sum_{j=1}^{n} p_j \left(f_{jk}^{+}\right)^2 \leq 0,$$

since $\lambda \leq \left[\sum_{j=1}^{n} p_j f_{jk}^{+}\right]^2 / \sum_{j=1}^{n} p_j \left(f_{jk}^{+}\right)^2$ for any solution $\varphi_k \in \Phi$ at $\lambda \in \left[0, \lambda^{*}\right]$.

As a corollary to the lemma, we obtain that$(1-\lambda)[B^{+}(p, \varphi_k)]^2 \leq \lambda \sigma^2(p, \varphi_k)$ at $\lambda \in \left[\lambda^{**}, 1\right]$, i.e., at these values λ, the modified criterion is more sensitive to the variance minimization criterion $\sigma^2(p, \varphi_k)$ than to the Bayesian criterion for maximizing the average gain.

If the value of $\lambda \in \left[\lambda^{*}, \lambda\right]$, then the values $\chi(p, \varphi_{ko})$ are sign-variable at $\varphi_k \in \Phi$, i.e., we cannot speak about the priority of the Bayes maximization criterion $B^{+}(p, \varphi_k)$ or the minimization criterion $\sigma^2(p, \varphi_k)$.

The following point estimates can be proposed for selection λ in the interval $\lambda \in \left[0, \lambda^{*}\right]$: $\hat{\lambda}_{\alpha}^{*}(p) = \left(\frac{n}{n-1}\right)^{\frac{\alpha}{2}} \lambda^{*} \rho^{\alpha}(p)$. Here, α is an arbitrary non-negative number; $\rho(p)$ is the distance from $p = (p_1, \ldots, p_n)$ to the midpoint $\left(\frac{1}{n}, \ldots, \frac{1}{n}\right)$ of the flat set $\Delta_n = \left\{p{:}0 \leq p_j \leq 1 \ (j = 1, \ldots, n),\ \sum_{j=1}^{n} p_j = 1\right\}$, equal to $\rho(p) = \left[\sum_{j=1}^{n}\left(p_j - \frac{1}{n}\right)^2\right]^{12} = \left(\sum_{j=1}^{n} p_j - \frac{1}{n}\right)^{12}$. The point estimates $\hat{\lambda}_{\alpha}^{*}(p)$ satisfy the following two axioms: 1) $\hat{\lambda}_{\alpha}^{*}(p^0) = 0$ at $\rho(p^0) = 0$, i.e., in the case of uniform distribution $p^0 = \left(\frac{1}{n}, \ldots, \frac{1}{n}\right)$, the modified criterion coincides with Bayes' criterion; 2) $\hat{\lambda}_{\alpha}^{*}(p^{*}) = \lambda^{*}$ at $\rho(p^{*}) = \max_{p\in\Delta_n}\rho(p) = \left(\frac{n-1}{n}\right)^{12}$, i.e., in the case of degenerate distribution p^{*} (one of the components of which is equal to one and the rest are zero), the variance $\sigma^2(p,\varphi_k) = 0$ for any $\varphi_k \in \Phi$.

Thus, if decision authority U believes that the value λ in the modified criterion $\chi(p,\varphi_{ko})$ satisfies the inequalities $0 \le \lambda \le \lambda^{*}$, then using the point estimate $\hat{\lambda}_{\alpha}^{*}$, a decision is made from the maximum $\chi(p,\varphi_{ko})$ over condition $\varphi_k \in \Phi$ for $\lambda = \hat{\lambda}_{\alpha}^{*}(p)$.

Partial cases of point estimates $\hat{\lambda}^{*}(p)$ at $\alpha = 0,1,2$ are the values of $\hat{\lambda}_0^{*}(p) = \lambda$, $\hat{\lambda}_1^{*}(p) = \sqrt{\frac{n}{n-1}}\rho(p)\lambda^{*}$, $\hat{\lambda}_2^{*}(p) = \frac{n}{n-1}\rho^2(p)\lambda^{*}$.

For selection $\lambda \in [\lambda^{**},1]$, are used point estimates of the form $\hat{\lambda}_{\alpha}^{**}(p) = 1 - \left(\frac{n}{n-1}\right)^2\rho^{\alpha}(p)(1-\lambda^{**})$ with non-negative α. The values $\hat{\lambda}_{\alpha}^{**}(p)$ satisfy the following two axioms: 1) $\hat{\lambda}_{\alpha}^{**}(p^0) = 1$ at $\rho(p^0) = 0$, i.e., in the case of uniform distribution $p^0 = \left(\frac{1}{n},\dots,\frac{1}{n}\right)$, the modified criterion coincides with the minimum variance criterion; 2) $\hat{\lambda}_{\alpha}^{**}(p*) = \lambda^{**}$ at $\rho(p^{*}) = \max_{p\in\Delta_n}\rho(p) = \left(\frac{n-1}{n}\right)^{12}$, i.e., in the case of degenerate distribution p^{*}, the variance $\sigma^2(p^{*},\varphi_k) = 0$ for any $\varphi_k \in \Phi$, and the optimal decision is made by Bayes' criterion.

Thus, if decision authority U believes that the value λ in the modified criterion $\chi_{\Delta}(p,\varphi_k)$ satisfies the inequalities $\lambda^{**} \le \lambda \le 1$, then using the point estimate $\hat{\lambda}_{\alpha}^{**}(p)$, a decision is made from the maximum $\chi(p,\varphi_k)$ over condition $\varphi_k \in \Phi$ for $\lambda = \hat{\lambda}_{\alpha}^{**}(p)$.

Particular cases of point estimates $\hat{\lambda}_{\alpha}^{**}(p)$ at $\alpha = 0,1,2$ are $\hat{\lambda}_{\alpha}^{**}(p^{*}) = \lambda^{**}$, $\hat{\lambda}_1^{**}(p) = 1 - \sqrt{\frac{n}{n-1}}\rho(p)(1-\lambda^{**})$, $\hat{\lambda}_2^{**}(p) = 1 - \frac{n}{n-1}\rho^2(p)(1-\lambda^{**})$.

The following point estimates can be suggested for selection $\lambda \in [\lambda^{*},\lambda^{**}]$: 1) $\hat{\lambda}_{\alpha}(p) = \lambda^{*} + \left(\frac{n}{n-1}\right)^{\frac{\alpha}{2}}\rho^{\alpha}(p)(\lambda^{**}-\lambda^{*})$, where $\alpha \ge 0$, and the point estimates satisfy the following two axioms: 1) $\hat{\lambda}_{\alpha}(p^0) = \lambda^{*}$ at $\rho(p^0) = 0$, i.e., in the case of uniform distribution $p^0 = \left(\frac{1}{n},\dots,\frac{1}{n}\right)$, in the modified criterion, the Bayes criterion is given greater preference; 2) $\hat{\lambda}_{\alpha}(p^{*}) = \lambda^{**}$ at $\rho(p^{*}) = \max_{p\in\Delta_n}\rho(p) = \left(\frac{n-1}{n}\right)^{12}$, i.e., in the modified criterion, greater preference is given to the criterion of minimum variance $\sigma^2(p,\varphi_k)$, and $\sigma^2(p^{*},\varphi_k) = 0$ for any $\varphi_k \in \Phi$, and the decision is made by the Bayes criterion.

Thus, if decision-making body U considers that the value of $\lambda \in [\lambda^{*},\lambda^{**}]$, then according to the modified criterion a decision is made from the condition of maximum $\chi(p,\varphi_k)$ at $\lambda = \hat{\lambda}_{\alpha}(p)$.

Particular cases of point estimates $\hat{\lambda}_{\alpha}(p)$ at $\alpha = 0,1,2$ are $\hat{\lambda}_0(p) = \lambda^{**}$, $\hat{\lambda}_1(p) = \lambda^{*} + \sqrt{\frac{n}{n-1}}\rho(p)(\lambda^{**}-\lambda^{*})$, $\hat{\lambda}_2(p) = \lambda^{*} + \frac{n}{n-1}\rho^2(p)(\lambda^{**}-\lambda^{*})$.

The derivation of the above point estimates is based on the use of the estimate $\beta + \gamma\rho^{\alpha}(p)(\delta_0 + \delta_1 + \delta_2\lambda^{**})$, whose coefficients are chosen in such a way that the above axioms are satisfied for each of the three cases of location.

7) *Conditional Decisions*. Let us compare the set K_I of previously considered decision-making criteria $K_I = \{\chi_1^1,...,\chi_1^{r_1}\}$ to the information situation I_1. From the set of decision criteria, the control body U selects one criterion, which is conditionally called the main criterion, and restrictions are imposed on the other decision criteria. Therefore, the decision made by the control body U according to the main criterion under given constraints on the other criteria from the set K_I, let us call it a *conditional decision*. Both for optimization problems and for decision-making, it is typical to set constraints either in the form of inequalities $c_1^l \le \chi_1^l \le C_1^l$, or in the form of equalities $\chi_1^l = c_1^l$.

It should be noted that since the search for an optimal solution is reduced to a finite number of options, setting a constraint in the form of an exact equality is in most cases not quite correct and leads to the absence of a conditional solution. In contrast, constraints in the form of inequalities are more natural and allow the decision-making body to conduct a kind of analysis to establish "reasonable" limits of c_1^l values and of C_1^l values from lower and upper limits of values of the criterion χ_1^l. For example, a book can be included in the LMAIA only if it mentions at least one Nobel Prize winner or member of the Nobel family (lower criterion).

Thus, if $\chi_1^s \in K_I$ is the main criterion, the conditional solutions are found from the following problem: $\chi_1^s(\varphi_{ko}) = \underset{\varphi_k \in \Phi}{\text{opt}}\, \chi_1^s(\varphi_k)$, $c_1^l \le \chi_1^l \le C_1^l$, $(l = 1,...,r;\ l \ne s)$. A special case of the formulated problem of finding conditional solutions is the case considering a subset $\overline{K_I} \subset K_I$ instead of a set K_I.

Example 1. *Let $\overline{K_I} = \{\chi_1^1, \chi_1^2\}$, where $\chi_1^1 = B^+(p, \varphi_k)$, $\chi_1^2 = \sigma^2(p, \varphi_k)$, and the vector of a priori distribution $p = (p_1,...,p_n)$ is given, and a χ_1^1 is the main criterion. The bounded solution φ_{ko} is found from the condition $B^+(p, \varphi_{ko}) = \max_{\varphi_k \in \Phi} B^+(p, \varphi_{ko})$, $c_1 \le \sigma^2(p, \phi_k) \le C_1$, where c_1 and C_1 are given positive constants. Note that it is possible to define the class of conditional solutions without distinguishing the main decision criterion directly as a solution of the system of inequalities $c_1^l \le \chi_1^l(\varphi_k) \le C_1^l$ $(l = 1,...,r_1)$.*

6. Conclusion

The obvious statement is that *decision-making in each static information situation* leads to the necessity to develop targeted methods depending on the considered criteria.

Thus, we have proved that it is possible to form a problem-oriented array under complex conditions of uncertainty by means of different variants of decision modeling and selection of an optimal model. Such an optimal model was tested on the local LMAIA on nobelistics of the International Nobel Information Center (INIC) and showed that it is possible to accurately and completely form the array under uncertainty.

This does not apply to the issues of decision-making in dynamics.

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