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Article

# Why the big bang never happened

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**Abstract:** If general relativity is correct, then the origin of the universe is a simple mathematical problem. The Friedmann equation in cosmology is a well-structured ordinary differential equation, and the global properties of its solutions can be qualitatively analyzed by the phase-trajectory method. In this paper we show that the total energy density of matter in the universe is positive, and the total pressure near the Big Bang is negative. By analyzing the global properties of the solutions to the Friedmann equation according to these two conditions of state functions, we find that the Big Bang is impossible, and the space must be a closed 3-dimensional sphere, the cosmological constant is likely to be zero, and the evolution of the universe should be cyclic. The analysis and the proof are simple and straight forward, therefore these conclusions should be reliable.

**Keywords:** space curvature; cosmological constant; negative pressure; big bang

## 1. Introduction

General relativity provides a scientific theoretical framework for the study of the universe. The discovery of Hubble's Law makes people realize that the whole universe also evolves dynamically. Astronomical observations suggest that the average distribution of galaxies over the region of $10^9$ light-years is roughly homogeneous and isotropic, and this observational fact is summarized as the cosmological principles. The main content of cosmology is built on the base of Einstein's field equations and cosmological principles. The universe is spherically symmetric at large scales, and hence the space-time metric is Friedmann-Lemaitre-Robertson-Walker (FLRW) one [1].

$$ds^2 = d\tau^2 - a(\tau)^2\left(dr^2 + \tilde{S}(r)^2(d\theta^2 + \sin^2\theta d\phi^2)\right),$$

in which $(\tilde{S}(r) = \sin r, r, \sinh r)$ correspond respectively to $(K=1,0,-1)$. $K$ represents the spatial curvature, $K = 1$ corresponds to the finite closed 3D sphere $S^3$, $K = -1$ to the open Lobachevskian space, while $K = 0$ corresponds to the critical flat Euclidean space $E^3$.

The FLRW metric contains only one undetermined scale factor $a(\tau)$. In 1922, A. Friedmann first derived the ordinary differential equation satisfied by $a$, that is, the Friedmann equation, which became one of the basic equations of cosmology. In cosmology, the non-zero term of the Einstein tensor can be easily computed by Maple [1,2].

$$\begin{cases} G_\tau^\tau = -\dfrac{3}{a^2}(a'^2 + K), a' = \dfrac{d}{d\tau}a(\tau) \\ G_r^r = G_\theta^\theta = G_\varphi^\varphi = -\dfrac{1}{a^2}(2aa'' + a'^2 + K) \end{cases} \tag{1}$$

The energy-momentum tensor (EMT) of all gravitational sources of matter can be merged into

$$T_\nu^\mu = \text{diag}(\rho_m, -P_m, -P_m, -P_m), \tag{2}$$

where $\rho_m$ is the total mass-energy density, and $P_m$ is called the total pressure, but in fact it contains the interaction potential between matter. The independent dynamic equations have only the following two

$$G_\tau^\tau + \Lambda + \kappa T_\tau^\tau = 0 \quad \Rightarrow \quad a'^2 = \frac{1}{3}\Lambda a^2 - K + \frac{\kappa}{3}\rho_{\rm m} a^2. \tag{3}$$

$$T_{;\nu}^{\mu\nu} = 0 \quad \Rightarrow \quad d(\rho_{\rm m} a^3) = -P_{\rm m} da^3, \tag{4}$$

where Λ is the cosmological constant, Equation (3) is the Friedmann equation. Equation (4) is the energy conservation law, which holds for each independent matter system [2]. If the equation of state $P_m = f(\rho_m, T, \cdots)$ is given, then the above system of equations is closed and its solution is unique under appropriate initial values.

On the theoretical aspect, all cosmological models of general relativity are solving Equations (3) and (4) under different state assumptions. For example, in the case of flat space dominated by dust, namely the Einstein—de Sitter universe [1], by $P_m = 0, K = 0$ we have solution $a \propto \sqrt[3]{\tau^2}$. Since the discovery of the accelerating expansion of the universe in 1998 through Ia type supernovae, dark matter and dark energy have become the new "two clouds" over the physical mansion, challenging the traditional standard models of particle physics and cosmology.

Typical dark matter and dark energy models include quintessence models [3], phantom model [4,5], etc. The quintessence models belong to the scalar field models, where a scalar field drives the cosmic acceleration of the universe [6–15]. Quintessence is particularly interesting as it represents the simplest scalar field dark energy scenario that avoids issues like ghosts or Laplacian instabilities. In quintessence models, the acceleration of the universe is driven by a slowly varying scalar field with a potential $V(\phi)$, similar to the mechanism of slow-roll inflation. However, in this case, contributions from non-relativistic matter, such as baryons and dark matter, are taken into account.

The description of dark matter and dark energy, usually using the equation of state $P = w\rho$ and $w = w(a)$, we obtain many specific models by fitting to the observed data [16–22]. Of course, the problem is far from being resolved [23]. In literature [20], by introducing the potential function $V(a)$ and transforming the Friedmann equation of some known dark energy models into Hamiltonian dynamics, the accelerating expansion of the universe is explained according to the evolutional trajectory. Some similar discussions for particular gravitational sources are provided in the literatures [24,25]. The cyclic universe model of nonlinear scalar field is discussed in reference [24]. In literature [25], the exact solutions for the ghost and electromagnetic fields are derived. In 2008, M. Novello and S. E. Perez Bergliaffa reviewed the general features of non-singular and cyclic universes, discussed the dynamical mechanisms behind the rebound, and analyzed examples of implementing these mechanisms [26].

The study of the universe has entered an era of precise cosmology. The Planck satellite has accurately measured small temperature fluctuations and polarization on the cosmic microwave background radiation (CMB) with unprecedented accuracy, providing strong limits on the cosmological parameters. In recent years, cosmology has also encountered very serious challenges. The main challenge is the obvious inconsistency between the observational data of the early universe and the late universe, such as the measurement of Hubble constant and matter density fluctuations. These conflicts suggest that the traditional Big Bang universe model contains internal contradictions and needs to be restudied. In fact, if the basic framework of general relativity is not problematic, then the origin of the universe is just a simple mathematical problem. Since the cosmic origin only involves the asymptotic behavior of the equation of state near the singularity, it has little relation with the specific matter model. In literature [27] the authors study similar problems, but are stuck in a mud pit of complex calculations. In this paper, under the most general equation of state of matter, we strictly prove that the cosmic initial singularities must not exist, and the universe is cyclic in time and closed and finite in space.

## 2. Energy-momentum tensor and function of state

By the derivatives of Equation (3) with respect to time $t$ and Equation (4), we get the second order differential equation

$$a'' = \frac{1}{6} a[2\Lambda - \kappa(\rho_m + 3P_m)]$$

Near the singularity of the Big Bang, the universe underwent inflation or accelerating expansion, that is, $a'' > 0$ when $a \to +0$, and thus we have

$$P_m < \frac{1}{3\kappa}(2\Lambda - \kappa\rho_m), (a \to +0) \tag{5}$$

Since $\Lambda$ is finite, and the mass-energy density of matter near the singularity is $\rho_m \propto a^{-3} \to +\infty$. From Equation (5) we learn, when the scale factor $a$ is small, there must be $P_m < 0$; that is to say, in the early universe with inflation, the total pressure of matter is certainly negative [19–22].

The mass-energy density of matter must be positive, otherwise some strange phenomena will occur, such as negative acceleration motion and repulsive gravity. For example, by Newtonian second law $F = ma$, the acceleration is in the opposite direction of force if $m < 0$, which is a conflicting phenomenon. The antiparticles only have the opposite charge rather than the negative mass. It can be shown that only the positive energy solution exists to the nonlinear spinor field [28]. Thus, only the total mass density $\rho_m \geq 0$ in Equation (2) is reasonable. Therefore, from the observations and phenomena we have conclusions,

Conclusion 1: The total mass-energy density of all matter in the universe is always positive

$$\rho_m > 0, (\forall a > +0) \tag{6}$$

In the early universe, the total pressure of all matter was negative

$$P_m < 0, (a \to +0) \tag{7}$$

Equations (6) and (7) are merely ordinary assumptions for the functions of state. However, if Einstein field equation is right, the two assumptions can actually provide explicit constraints on the structure and evolution of the universe [2].

In the next, we use several usual kinds of matter as examples to further clarify the rationality of the above hypothesis. Due to the requirement of covariance, the matter fields can be described only by the scalar and spinor fields, while other interaction potentials except for the gravitational field can only be described by the 4-dimensional vector potentials. The Lagrangian of a composite physical system is a superposable real scalar, which is the sum of the Lagrangians of all subsystems and the interaction potentials. Thus, the total Lagrangian of all matter in cosmology must have the following form [28]

$$\bar{L} = \frac{1}{2\kappa}(R - 2\Lambda) + \bar{L}_m, \bar{L}_m = \bar{L}_\phi + \bar{L}_p + \bar{L}_s + \bar{L}_A + \bar{L}_\Phi + \cdots \quad (8)$$

in which $\kappa = 8\pi G$, $R$ is the scalar curvature, $\bar{L}_m$ is the total Lagrangian of matter, $\phi$ is the slow-rolling scalar field

$$\bar{L}_\phi = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi), V\sim\frac{m^2}{2k}\phi^{2k}(|\phi| \to \infty) \quad (9)$$

In this paper, the Einstein summation convention is used, the Greek alphabet represents the 4-dimensional space-time index, and $(j,k,l)$ represents the space coordinate index. $\bar{L}_p$ corresponds to the classical dust or perfect fluid

$$\bar{L}_p = -\sum_n m_n\sqrt{1 - v_n^2}\delta^3(\vec{x} - \vec{X}_n), v_n^2 = g^{00}\bar{g}_{kl}\frac{dX_n^k}{dt}\frac{dX_n^l}{dt}$$

$\bar{L}_s$ corresponds to the nonlinear spinors

$$\bar{L}_s = \sum_n (\Re\langle\psi_n^+\alpha^\mu\widehat{p}_{n|\mu}\psi_n\rangle - \psi_n^+\Omega_\mu\widehat{S}^\mu\psi_n - m_n\psi_n^+\gamma^0\psi_n + N_n)$$

$$\bar{L}_A = -\frac{1}{2}\nabla_\mu A_\nu\nabla^\mu A^\nu, L_\Phi = \frac{1}{2}(\nabla_\mu\Phi_\nu\nabla^\mu\Phi^\nu - b^2\Phi_\mu\Phi^\mu), \cdots$$

in which $A^\mu$ represents the electromagnetic interaction potential, and $\Phi^\mu$ represents the short-distance strong interaction potential

$$\hat{p}_k^\mu = i(\hbar\partial^\mu + \Upsilon^\mu) - e_k A^\mu - s_k\Phi^\mu, e_k = \{0, \pm e\}, s_k = \{0, s\},$$

where $\alpha^\mu$ is the current, and $\hat{S}^\mu$ is the spin matrices.

By the variation of Lagrangian (8) with respect to the metric $g_{\mu\nu}$, we get Einstein field equation

$$G^{\mu\nu} + \Lambda g^{\mu\nu} + \kappa T^{\mu\nu} = 0, G^{\mu\nu} = -\frac{\delta(R\sqrt{g})}{\sqrt{g}\delta g_{\mu\nu}} = R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R \quad (10)$$

in which $\frac{\delta}{\delta g_{\mu\nu}}$ is the Euler derivatives, $T^{\mu\nu}$ is the EMT of matter

$$T^{\mu\nu} = -2\frac{\delta(\bar{L}_m\sqrt{g})}{\sqrt{g}\delta g_{\mu\nu}} = -2\left(\frac{\partial\bar{L}_m}{\partial g_{\mu\nu}} - (\partial_\alpha + \Gamma_{\alpha\gamma}^\gamma)\frac{\partial\bar{L}_m}{\partial(\partial_\alpha g_{\mu\nu})}\right) - g^{\mu\nu}\bar{L}_m$$

By detailed calculation we have the EMT for each component [1,2]

$$T^{\mu\nu} = T_\phi^{\mu\nu} + T_p^{\mu\nu} + T_s^{\mu\nu} + T_A^{\mu\nu} + T_\Phi^{\mu\nu} + \cdots \quad (11)$$

$$T_\phi^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - V(\phi)] \quad (12)$$

$$T_p^{\mu\nu} = \sum_n m_n u_n^\mu u_n^\nu\sqrt{1 - v_n^2}\delta^3(\vec{x} - \vec{X}_n), \quad u_n^\mu = \frac{d}{ds}X_n^\mu \quad (13)$$

as well as

$$T_s^{\mu\nu} = \sum_n \left(\frac{1}{2}\Re\langle\psi_n^+(\alpha^\mu\hat{p}_n^\nu + \alpha^\nu\hat{p}_n^\mu)\psi_n\rangle + g^{\mu\nu}N_n\right) \quad (14)$$

$$T_A^{\mu\nu} = -\nabla^\mu A^\alpha\nabla^\nu A_\alpha + \frac{1}{2}g^{\mu\nu}\nabla_\beta A_\alpha\nabla^\beta A^\alpha \quad (15)$$

$$T_{\Phi}^{\mu\nu}=\nabla^{\mu}\Phi^{\alpha}\nabla^{\nu}\Phi_{\alpha}-\frac{1}{2}g^{\mu\nu}\nabla_{\beta}\Phi_{\alpha}\nabla^{\beta}\Phi^{\alpha}+\frac{1}{2}b^{2}(\Phi^{\mu}\Phi^{\nu}+g^{\mu\nu}\Phi_{\alpha}\Phi^{\alpha}) \quad (16)$$

For Equation (13), denote

$$\bar{m}\equiv\frac{1}{N}\sum_{n=1}^{N}m_{n},\bar{\rho}\equiv\frac{N\bar{m}}{V}=\frac{\varrho}{a^{3}},J=\frac{kT}{\bar{m}c^{2}}$$

in which $\varrho$ is the conformal mass density of proper mass, $J$ is the dimensionless temperature. The following conclusion can be proved by introducing the driving effect of the gravitational potential [2,29].

Conclusion 2: The equation of state of perfect fluid compatible with relativity should be

$$P_{p}=\frac{\bar{\rho}kT}{\bar{m}c^{2}}\left(1-\frac{kT}{2(\sigma\bar{m}c^{2}+kT)}\right)$$

The functions of state of an adiabatic process satisfy

$$\bar{\rho}=\rho_{0}[J(J+2\sigma)]^{\frac{3}{2}},\rho_{p}=\bar{\rho}(1+\frac{3}{2}J),P_{p}=\bar{\rho}\frac{J(J+2\sigma)}{2(J+\sigma)} \quad (17)$$

where $J\to a^{-1},(a\to 0)$ acts as parameter, $\rho_{0}=\varrho(\sigma b)^{-3}$ is a constant in cosmology with a mass density dimension.

Obviously, the pressure of perfect fluid is always positive, and $P_{p}\propto a^{-4}$ if $a\to +0$. Vector fields such as electromagnetic fields have two different types of EMT, one is the moving photons, whose EMT is also in the form of Equation (13) and can be incorporated into the EMT of classical dust. Another type is the static electromagnetic field of charged particles, which can be incorporated into the EMT of spinors [2]. By the classical approximation of (14) we have

$$\psi^{+}\alpha^{\nu}\psi\to u^{\nu}\sqrt{1-v^{2}}\delta^{3}(\vec{x}-\vec{X}),\hat{p}^{\mu}\psi\to mu^{\mu}\psi,N\to w\sqrt{1-v^{2}}\delta^{3}(\vec{x}-\vec{X}) \quad (18)$$

Substituting Equation (18) into Equation (14), we get the classical approximation of the total EMT of spinors and their interaction potentials as follows

$$T_{s}^{\mu\nu}\to\sum_{n}\left(m_{n}u_{n}^{\mu}u_{n}^{\nu}+w_{n}g^{\mu\nu}\right)\sqrt{1-v_{n}^{2}}\delta^{3}(\vec{x}-\vec{X}_{n}) \quad (19)$$

where $w_{n}\propto(\sqrt{\bar{g}})^{-1}>0$ is equivalent to a negative pressure, taking the same role as the cosmological constant $\Lambda$ in Einstein field Equation (10). Like the case of perfect fluid, making statistical average of Equation (19) we have.

Conclusion 3: The EMT of nonlinear spinor gas with interaction potentials is given by

$$T_{s}^{\mu\nu}\to(\rho_{s}+P_{s})U^{\mu}U^{\nu}+(W_{s}-P_{s})g^{\mu\nu} \quad (20)$$

in which $\rho_{s}$ and $P_{s}$ have properties similar to those of the perfect fluid. The function of state $W_{s}$ has the effect of negative pressure, and $W_{s}(a)$ takes the position of the cosmological constant $\Lambda$ in Einstein field equation, thus it may be the physical origin of the cosmological 'constant' $\Lambda$.

For a spinor with nonlinear potential, the equivalence principle holds approximately, so the functions of state $(\rho_{s},P_{s})$ are approximately equal to $(\rho_{p},P_{p})$ of the perfect fluid. By Equation (19), the microscopic $W_{s}$ is defined as follows [2]

$$W_{s}\equiv\frac{1}{V}\int_{V}\sum_{n}w_{n}\,\delta^{3}(\vec{x}-\vec{X}_{n})\sqrt{1-v_{n}^{2}}dV=\frac{1}{V}\sum_{X_{n}\in V}w_{n}\sqrt{1-v_{n}^{2}} \quad (21)$$

Denote the mean parameters by

$$\bar{w}=\frac{1}{N}\sum_{n=1}^{N}w_n\to\frac{C_1}{a^3},\mu=\frac{1}{N}\sum_{n=1}^{N}\frac{w_n}{m_n}\to\frac{C_2}{a^3},(a\to+0)$$

By the classical approximation of Equation (17) and Equation (21) we have

$$W_s=\bar{\rho}\left(\frac{\bar{w}}{\bar{m}}-\frac{3\mu\sigma J}{2(J+\sigma)}\right)=\frac{\varrho}{a^3}\left(\frac{\bar{w}}{\bar{m}}-\frac{3\mu\sigma(\sqrt{a^2+b^2}-a)}{2\sqrt{a^2+b^2}}\right)\to\frac{C}{a^6}$$

Thus we have

Conclusion 4: For nonlinear spinors, by Equation (20) we obtain

$$T^{\tau}_{\tau|s}=\rho_s+W_s,T^{r}_{r|s}=T^{\theta}_{\theta|s}=T^{\varphi}_{\varphi|s}=W_s-P_s$$

So the equation of state in cosmology is equivalent to

$$w_s(a)=\frac{P_s-W_s}{\rho_s+W_s}\to-1,(a\to+0)$$

in which $W_s$ is equivalent to negative pressure, $W_s\propto a^{-6}$ is the dominant term in total pressure $P_m$ when $a\to+0$. The nonlinear dark spinors determine the initial state and the large-scale structure of the universe. $W_s$ acts like both the mass-energy density of dark matter and the negative pressure of dark energy.

According to the conservation laws of energy and momentum $T^{\mu\nu}_{;\nu}=0$ of Equation (20), we obtain the equation of motion for the dark spinor gas

$$(\rho_s+P_s)U^\nu U^\mu_{;\nu}=(g^{\mu\nu}-U^\mu U^\nu)\partial_\nu(P_s-W_s) \tag{22}$$

Equation (22) is quite different from the geodesic equation $U^\nu U^\mu_{;\nu}=0$, so the dark halo in a galaxy is automatically separated from ordinary matter during the formation of the galaxy.

In cosmology, although we call $P_m$ the total pressure, it is actually a function of state that includes all the interaction potentials, so the negative pressure is reasonable [2,19]. Except for the global scalar field $\phi$, all other matter fields have only very tiny local structures, and the classical approximation of EMT has the standard form Equation (20), so the equations of state are all algebraic equations. For the scalar field $\phi$, the equation of state is a differential equation, so the asymptotic solution near the singularity should be calculated. By Equation (9) and Equation (12) we have

$$\bar{L}_\phi=\frac{1}{2}\phi'^2-V,\rho_\phi=\frac{1}{2}\phi'^2+V,P_\phi=\frac{1}{2}\phi'^2-V \tag{23}$$

If $a\to+0$, we have $|\phi|\to\infty$, so we only need to consider the highest order term in potentials $V(\phi)$. In the case of $V=\frac{1}{2k}m^2\phi^{2k},\quad(k\geq1)$, by variation of $\phi$ we have dynamic equation

$$\phi''+\frac{3a'}{a}\phi'+m^2\phi^{2k-1}=0 \tag{24}$$

If the Big Bang exists, we set $a(0)=0$. For zero-pressure dust we have $a\propto\sqrt[3]{\tau^2}$, so without loss of generality we have

$$a\to a_0\tau^j,\phi\to\frac{C_0}{\tau^i},(j\geq\frac{2}{3},i>0) \tag{25}$$

Substituting Equation (25) into Equation (24) we obtain the solution for $k>1$

$$i=\frac{1}{k-1},m^2=\frac{3j(k-1)-k}{C_0^{2(k-1)}(k-1)^2},j>\frac{k}{3(k-1)} \tag{26}$$

$$\rho_\phi \to \frac{3jC_0^2}{2k(k-1)}\tau^{-\frac{2k}{k-1}}, P_\phi \to -\frac{3j(k-1)-2k}{2k(k-1)^2}C_0^2\tau^{-\frac{2k}{k-1}} \tag{27}$$

Substituting $a \to a_0\tau^j$ into Equation (27) we obtain

$$P_\phi \to \left(\frac{2k}{3(k-1)} - j\right)\frac{C^2}{a^n}, (a \to +0) \tag{28}$$

where $n = \frac{2k}{j(k-1)}$. By Equation (28) we find that, if $j > \frac{2k}{3(k-1)}$ then the nonlinear scalar field in the neighborhood of singularity can provide negative pressure and $0 < n < 3$. Substituting $\rho_\phi \to C^2 a^{-n}$ into Friedmann Equation (3) we have

$$a'^2 \geq C_1^2 a^{2-n} \Rightarrow a \geq a_0\tau^{\frac{2}{n}}, (a \to +0)$$

In the case of $k = 1$, Equation (24) is a linear equation of $\phi$, and the solution is an oscillating function of $\tau$. For $\tau \to +0$ we have asymptotic solution $\phi \to C\, ln\, \tau$, so $|\phi'|$ is a higher order infinity of $|\phi|$. Substituting it into Equation (23) we obtain

$$\rho_\phi \doteq P_\phi \to \frac{C_2^2}{\tau^2} > 0, (a \to +0)$$

It shows that the linear scalar $\phi$ cannot provide initial negative pressure and the cosmic inflation. To sum up, we have:

Conclusion 5: Both the nonlinear spinors and the nonlinear scalar field can provide the initial negative pressure. When $a \to +0$, we have

$$P_\phi \propto -a^{-n}(0 < n < 3), P_p \propto a^{-4}, P_s \propto -a^{-6} \tag{29}$$

The total pressure is controlled by $P_s$, and the nonlinear spinors can provide the driving force for the accelerating expansion of the universe. Thus, the conclusion 1 is not only an empirical fact but also has a profound theoretical background.

## 3. The big bang is impossible

Since the Friedmann equation is an ordinary differential equation with good structure, the global properties of the solutions can be qualitatively analyzed by phase-trajectory method. In literatures [2,20], under the different assumptions, the dynamical behavior of the universe is analyzed by means of Hamiltonian formalism of the Friedmann equation. In the next, we qualitatively analyzed the overall properties of the solution to Friedmann equation according to the matter state Assumptions (6) and (7). We find that the universe cannot reach the initial singularity and we have the parameters $K = 1$ and $\Lambda \cong 0$. That is, the spatial structure of the universe is a closed 3-dimensional sphere $S^3$ and the cosmological constant is likely zero, in addition, the evolution of the universe should be cyclic in time.

In the above analysis, the scale factor $a(\tau) \sim O(\tau^j)$ is not analytic at the origin, which increases the difficulty of the analysis. For example, in literature [1] the scale factor of the closed or open universe of zero-pressure dust can only be expressed by parametric equations. If the following conformal FLRW metric is adopted, then the solution of the equation is analytic, and the solution corresponding to the zero-pressure dust is an elementary function.

$$ds^2 = a(t)^2\left(dt^2 - dr^2 - \tilde{S}(r)^2(d\theta^2 + \sin^2\theta d\phi^2)\right).$$

in which $d\tau = a(t)dt$, and the Friedman Equation (3) becomes

$$\dot{a}^2 = -Ka^2 + \frac{1}{3}\Lambda a^4 + \frac{1}{3}\kappa\rho_m a^4, \dot{a} = \frac{d}{dt}a(t) \tag{30}$$

For convenience, we make some transformations for the variables in Equation (30). Define the total conformal proper mass-energy density by

$$\bar{R} \equiv \lim_{a\to\infty}\frac{1}{6}\kappa\rho_m a^3 \approx \frac{1}{6}\kappa\varrho \tag{31}$$

Make transformation for the total mass density by

$$\rho_m = \frac{3}{\kappa a^3}\left(2\bar{R} + \frac{X(a)}{a}\right) \tag{32}$$

in which $X(a)$ corresponds to the unknown part in $\rho_m$. By Equation (31) we have $X(a)/a \to 0$ as $a \to \infty$. The specific form of $X(a)$ is not important for the following qualitative analysis, only its asymptotic nature as $a \to +0$ has influences on the conclusion. Equation (30) can be rewritten as

$$\dot{a}^2 = F(a), F(a) \equiv 2\bar{R}a - Ka^2 + \frac{1}{3}\Lambda a^4 + X(a) \tag{33}$$

From Equation (33) we find that $\bar{R}$ has length dimension, which represents the average scale factor of the universe. The Friedman equation in the form of Equation (33) is more convenient to discuss the properties of its solution in detail by the phase-trajectory method. Comparing Equation (33) with Equation (30), we get the following relation [2]

$$\rho_m = \frac{3}{\kappa a^4}\left(F(a) + Ka^2 - \frac{1}{3}\Lambda a^4\right) \tag{34}$$

Substituting Equation (32) into the energy conservation law (4), we obtain the total pressure

$$P_m = -\frac{1}{\kappa a^4}[X'(a)a - X(a)] \tag{35}$$

$P_m$ is only determined by $X$ but independent of $\bar{R}$. By Equation (35) and Equation (29) we find that $X(a) \propto -a^{-2}$ as $a \to +0$. Since the derivative of pressure and potential corresponds to the ordinary force which must be finite, we have at least $X(a) \in C^1$, i.e., $P_m$ should be at least continuous. By Equation (35) we have $X(a) \in C^1$. Again by the definition of $F(a)$ in Equation (33), we also have $F(a) \in C^1$.

According to the Assumptions (34) and (7), i.e., the positive total mass energy density $\rho_m > 0$ and negative initial pressure $P_m(a) < 0, \quad (a \to +0)$, by Friedmann Equation (33) we have

Conclusion 6: for $F(a) \in C^1$, if $\lim_{a\to 0} X(a) \neq 0$ and the condition (7) holds, then we have

$$F(a) < 0, (a \to +0) \tag{36}$$

Proof In the case of $0 < |X(0)| < \infty$, by Equation (35) and Equation (7) we have

$$P_m \to \frac{X(0)}{\kappa a^4} < 0, (a \to +0)$$

By the definition of $F(a)$ in Equation (33), we have

$$F(0) = X(0) < 0$$

In the case of $X \to X_0 a^{-n}, (a \to +0, n > 0)$, again by Equation (35) and Equation (7) we have

$$P_m \to \frac{(n+1)X_0}{\kappa a^{4+n}}, (a \to +0)$$

By $P_m < 0$ we find $X_0 < 0$. According to the definition of $F(a)$ in Equation (33) we have

$$F \to \frac{X_0}{a^n} < 0, (a \to +0)$$

So we prove that Equation (36) holds for all cases.

The following important inference is drawn from the above conclusions.

Conclusion 7: The evolution of the universe cannot reach the initial singularity, that is, the Big Bang could not happen, we always have $a(t) > 0$.

Proof For the solution of the Friedmann Equation (33), we have $F(a) = \dot{a}^2 \geq 0$. From the continuity of (36) and $F(a)$, the equation $F(a) = 0$ must have a positive root $0 < a_0 \ll \bar{R}$. If $F(a) = 0$ has only this positive root $a_0$, by $F(a) \in C^1$ we find that $F(a)$ can be expressed in the following form

$$F(a) = (a - a_0)A(a), \qquad (A > 0, \quad \forall a \geq 0). \tag{37}$$

If $F(a) = 0$ has a series of positive roots $0 < a_0 < a_1 < a_2 < \cdots$, for realistic universe $F(a)$ must be single-connected curves, and thus it can be expressed as

$$F(a) = (a - a_0)(a_1 - a)B(a), \qquad (B > 0, \quad 0 \leq a \leq a_1). \tag{38}$$

Since the Friedmann equation is an equation in the average sense, the multiple roots are meaningless in physics.

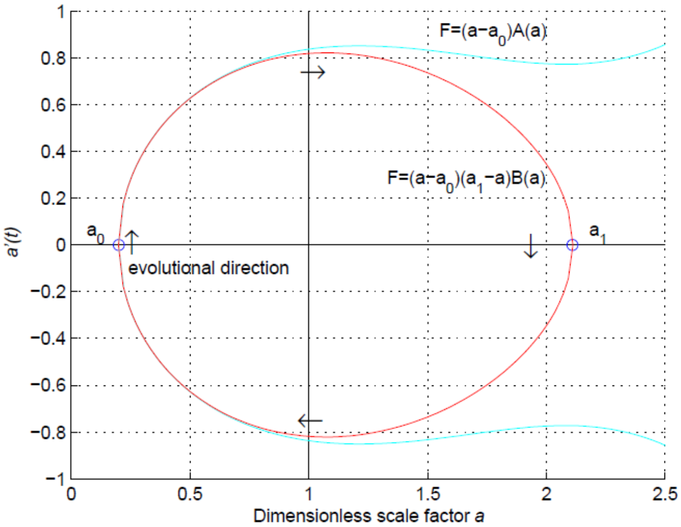


**Figure 1.** The phase trajectory $a \sim \dot{a}$ of the Friedmann Equation (33).

The connected phase-trajectory $a \sim \dot{a}$ of dynamic Equation (33) under conditions (37) or (38) is illustrated by **Figure 1**, in which we have set the average scale $\bar{R} = 1$. Equation (38) corresponds to the cyclic cosmic model, while Equation (37) corresponds to the bounce cosmic model. We set the time origin $t = 0$ at the turning point $a(t) = a_0$. The **Figure 1** clearly shows that $a(t)$ cannot reach the origin 0, therefore the initial singularity is absent.

At $a(t) = a_0$, by (37) or (38) we have $F(a_0) = 0$. Substituting it into Equation (34) we obtain

$$0 < \rho_m(a_0) = \frac{3}{\kappa a_0^2}\left(K - \frac{1}{3}\Lambda a_0^2\right) \tag{39}$$

Since $\Lambda \geq 0$ in cosmology, by Equation (39) we certainly have $K = 1$. Thus we get another important inference.

Conclusion 8: The spatial curvature $K = 1$, and the cosmic space compatible with the total mass-energy density $\rho_m > 0$ is a closed 3-dimensional sphere $S^3$.

For the cyclic closed cosmic model (38), we have an estimate of the upper bound of the cosmological constant Λ. Substituting Equation (38) into Equation (34) and letting $a = a_1$, by Equation (34) we have

$$0 < \rho_m(a_1) = \frac{3}{\kappa a_1^2}\left(1 - \frac{1}{3}\Lambda a_1^2\right)$$

Clearly $\bar{R}a$ constitutes the main part of the mass-energy density as $\to a_1$ , and $|X(a)| \ll \bar{R}a$ can be omitted. By $\Lambda \geq 0$ and $a_1 \geq 2\bar{R}$, according to observational data, we have the following estimation [2]

$$0 \leq \Lambda < \frac{3}{a_1^2} \leq \frac{3}{4\bar{R}^2} \sim 10^{-24}\text{ly}^{-2} \tag{40}$$

Thus, for the cosmological constant Λ in the cyclic closed universe, we obtain the inference.

Conclusion 9: When $a \to +0$, the cosmological term $\Lambda a^4$ is a high-order infinitesimal, which has no substantial effect on the behavior of the early universe, and Λ is zero or a very tiny positive number.

The estimate (40) is less than the current observed data, and this difference can be explained by the state function $W(a)$ in EMT, which is equivalent to a decaying $\Lambda(t)$ in Friedmann equation. Thus setting $\Lambda = 0$ is a good choice in cosmology.

If Λ is large enough, when $a \to \infty$, the behavior of the Fredmann Equation (33) is controlled by the dominant term $\Lambda a^4$, and the cosmological model is bouncing. The global solution of the Friedmann equation exists, we have

$$\left(\frac{da}{d\tau}\right)^2 \to C^2 a^2 \Rightarrow a \to C_0 \cosh(C\tau)$$

However, the bouncing model is inconsistent with the isotropy and homogeneity of the current universe, because there is no causal correlation between distant regions before the turning point $t < 0$, the universe should be severely anisotropic and inhomogeneous, and some information should be preserved. This problem is absent in a cyclic closed universe because anisotropy and inhomogeneity in a period will be polished by subsequent evolution until the increasing entropy is balanced with the interaction between matter. From the above analysis, we find that the cosmological term $\Lambda a^4$ is purely a troublesome term which has not any practical use. For the ripping model, the Friedmann equation requires a higher order energy term than $\Lambda a^4$, which is physically unreasonable.

By $K = 1, \Lambda = 0$ and Equation (33), we obtain the following cosmic dynamic equation with geometrized parameters.

Conclusion 10: The universe with nonlinear spinors satisfies the cyclic closed cosmological model, and the Friedmann equation is equivalent to

$$\dot{a}^2 = \frac{1}{a^2}(a - \alpha)(\omega - a)(a^2 + 2\delta a + \varepsilon^2), (\alpha, \omega, \delta, \varepsilon \geq 0) \tag{41}$$

where $\alpha > 0$ is the minimum scale factor, while $\omega$ is the maximum scale factor, and $(\varepsilon, \delta \geq 0)$ are two small parameters reflecting the influence of dark energy. $(\varepsilon, \delta)$ all have length dimensions, and they can be determined by fitting the observational data. For the realistic universe, we have

$$\alpha, \delta, \varepsilon \ll \omega.$$

The solution of Equation (41) is elliptic functions, whose properties have been clearly studied. In the previous researches, the cosmological model is usually obtained from the assumptions on the equations of state of matter and then fitted the observed data, which is like looking for a needle in a haystack. A more reasonable approach would be to assume the algebraic form of the right-hand term in Equation (41) by fitting the astronomical observation data, and then to conversely determine the functions of state of matter, because the right-hand term is relatively simpler in form with little selection, and the parameters are easier to be determined [2].

## 4. Conclusions

The common view is that the singularity of space-time is a very complicated problem. In fact, the really complicated problem is that several comparable variables nonlinearly couple together to produce the most complex phenomenon, just like the hydrodynamics. Mathematically, the singularity usually simplifies the problem, like Dirac-δ and the point charges. The above analysis shows that the functions and equation of state near the Big Bang singularity become very simple, only needing to consider the main term. In literature [27], the author also analyzed the equations of state near the Big Bang, and calculate the EMT of various kinds of matter, such as slow-rolling scalar field, magnetic monopolar topological soliton, the grand unified theory $SU(5)$, gauge field symmetry breaking, quantum field theory and renormalization diagram, effective temperature quantum field theory, Feynman path integral, instanton solution, quantum fluctuation, supersymmetry theory, chaos inflation and soon. Such treatment complicates the problem and cannot solve the origin of the universe, but makes more confusion and puzzles. Although only the equations of state and the properties of scalar field, spinors and perfect fluid are analyzed above, it is easy to understand the general validity of this method, the clarity and reliability of the conclusions. This also reflects the power of abstraction and logic, just like the palm of the Buddha, we need not to tangle too many details.

Literature [30] reviews the singularity in general relativity and also complicates the problem. The first sentence of the article asserts that "in the framework of general relativity, the singularity of space-time is inevitable". However, the cosmological model discussed above is obviously singularity-free. As the simplest Schwarzschild solution, you will be puzzled if only take the horizon $r = 2m$ as a static singularity. In literature [30] some coordinate transformations to remove the singularity are introduced, but they are not true, because the legitimate coordinate transformation must have suitable smoothness on the entire manifold. For example, the Kruskal transformation, the Eddington-Finkelstein transformation, etc., even the first derivative is discontinuous at $r = 2m$, so such transformations are only locally valid. The coordinate transformation is a properly smooth bijection, and these coordinate transformations can be used either for outside the horizon or for inside the horizon, but they just cannot cross the horizon.

To understand the physical singularity, you cannot stand within the singularity. It should be understood as a changing process like the treatment of infinity or limits in mathematics. The above process of analyzing the Big Bang singularity in this paper is a concrete example. If you take the Schwarzschild black hole as an evolving result from a dense star without singularity to collapse into a black hole, you will see clearly the essence of the problem. In fact, the temporal singularity and spatial singularity of the star occurs at different times and in different locations. The analysis in paper [29] shows that the temporal singularity first occurs in the center, and in this case the particles near the center of the star begin to escape at the speed of light, and the central region begins to transpose the time and radius. Then the mass-density at the horizon tends to infinity, and inside and outside the horizon "Yin and Yang are separated". At $r = 2m$, there is a concentrated mass distribution, and the particles move at the speed of light in the horizon. Because the EMT contains the momentum of the particles, the velocity of the collapsing particles approaches the speed of light, and the escaping particles cannot be accumulated at the center of the star. The equation of state compatible with relativity must reckon in the driving effect of the gravitational potential. The calculation in paper [29] shows that there is singularity-free star with any given large mass and it does not need thermonuclear reaction to balance the gravity.

There are so many counterexamples for "singularity theorem", except for the ambiguous concepts, because some conditions are invalid in the realistic world. For example, for the energy condition of the singularity theorem, the influence of negative pressure or potential is ignored, so the energy condition is not generally true. Second, Einstein field equation clearly includes the motion of the particles, so the closed trapped surface cannot be formed dynamically, and the particles in a star cannot stay and accumulate near the center when the space is severely curved. In the language of gravity, the gravitational field is a conservative field, and the sum of the total kinetic energy and gravitational potential energy of the particles is conserved, so you cannot concentrate all the matter within the event horizon. Third, there is only one realistic simultaneous hypersurface in the universe [31], but the derivation of the Raychaudhuri equation unconsciously uses the future properties of the space-time, which is illegal in dynamical analysis. Therefore, the singularity theorem proved by using this equation is also invalid. In the study of physical problems, we should not only have a clear understanding of the connotation of physics, but also have a deep understanding of mathematical methods, so as not to be lost in the complicated representation.

There is a point of view that General Relativity does not hold below the Planck length, and hence the whole argumentation may break down, that is, one cannot take $a \to 0$ in General Relativity, and the Friedmann equations may need to be corrected there. However, an infinitesimal is not a concrete number, but a variable for analysis. The earth, for example, is an infinitesimal size in the solar system, but it is actually very large. Although the above analysis uses $a \to 0$, the minimum scale α of the universe is still very large. If analysis cannot be used under Planck length, then all physical dynamic equations, such as Einstein field equation and Dirac equation, will be invalid, because the partial differentials are defined upon the limit $\Delta x \to 0$.

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