Mathematics and Systems Science (2023) Volume 1 Issue 1
doi: 10.54517/mss.v1i1.2290

Review Article

Developments and progresses of meshing theory for gearing

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ABSTRACT

Gear drive dominates in mechanical transmissions and has comprehensive advantages of high precision, high efficiency, high reliability, long life and the ability to realize large drive ratios. The main purpose of this article is to review research development in meshing theory, which involves the meshing theory of the one degree-of-freedom (1DOF) line-conjugate tooth surface couple, the 1DOF point-conjugate tooth surface couple and the two degree-of freedom (2DOF) conjugate tooth surface couple. Some compendious discussions are made on the significant results and progresses in meshing theory developments of gearing.

Keywords: gear drive; meshing theory; research developments and progresses

1. Introduction

The gear mechanism can transmit rotary motion between parallel, intersecting and staggered axes[1]. In the other words, it can transmit rotary motion between two axes in any configuration. For a long time, people's thinking has been limited to the cognitive involute cylindrical gear drive. Yet, there are many gearing forms, such as spur gear pair, helical gear pair, bevel gear pair and worm gear pair, etc. Although gearing has been invented for a long time, tooth profiles of the gear for drive motions has been really studied since 17th century. Until now, it is mainly used in the gear mechanism with a constant transmitting ratio between the driving member and the driven member. Recently, non-circular gears have been utilized for transmission of rotary motion between two components with variable transmission ratios[2]. The development of gearing type will be promoted by improving the load carrying capacity, gear cutting efficiency, manufacturing accuracy and various invention patents of the gear drive.

Meshing theory for gear drives is also called conjugate surface theory, is used to study the meshing transmission of higher pair mechanisms. It is a branch of science related to differential geometry, manufacturing, design, metrology and computerized methods of investigation[3]. Meshing theory for gear drives is continuously developed and applied. This has made possible the scientific design of gearing and the innovation of its manufacturing technology. It is not difficult to notice that during the operation of a gearing system, the transfer of motion and force is achieved by the continuous contact of the tooth surfaces. So two meshing tooth surfaces are conjugate surfaces, and the meshing theory was taken as a systematic research on gear drives from this perspective. All of this requires meshing theory knowledge: researching new forms of meshing transmissions, evaluating gear quality, developing new methods to manufacture gears, analyzing

ARTICLE INFO
Received: 29 August 2023 | Accepted: 18 October 2023 | Available online: 3 January 2024

CITATION
Zhao Y, Ma J. Developments and progresses of meshing theory for gearing. *Mathematics and Systems Science* 2023; 1(1): 2290. doi: 10.54517/mss.v1i1.2290

meshing accuracy of gears and designing the profile of generating tools for machining gears.

2. History of developments

Meshing theory for gear drives began with the research on cycloidal gear by Camus[1] in the early 18th century, and the basis law of the tooth profile engagement was proposed. In the 1860s, the involute cylindrical gear engagement was proposed by Swiss scholar Euler[2], and the mathematical basis of involute tooth profiles was analyzed and studied. Moreover, the positional relationship between the gear profile's radius of curvature and center of curvature was illustrated. Subsequently, Euler's work was generalized by Savary, and sort out the Euler-Savary formula[3]. Entering the 19th century, the relationship between the principal curvature and the geodesic torsion for a point on a surface was discovered by Bertrand (1822–1900), and the corresponding formulas were derived by him[1]. Based on the above research work, a mathematical foundation for the curvature of the tooth surface was provided. Willis[4] proposed the famous fundamental law for the tooth profile engagement of the planar gear pair, i.e., the Willis theorem, and authored "Principles of mechanisms" in 1841. The enveloping surface method (a classical method of solving conjugate tooth surfaces) was first proposed by the French geometrician T.Oliver[2]. But, its role of analytical method in meshing principle was not recognized at that time, and the meshing problem was totally attributed to a problem of descriptive geometry. The Russian mathematician Gochman[2] introduced analysis method in study of the meshing theory, and obtained the same result as T.Oliver's earlier studies. Thus, analysis method of meshing theory was established. Until 1943, the kinematic method of spatial engagement has just been gradually developed. Based on Gochman's theory, Dudley and Poritsky[5], Poritsky[6] and other researchers developed this technique, which established the foundation for meshing theory in contemporary gears.

Through nearly 200 years development, several branches have been formed, as follows: the meshing theory of the 1DOF (one degree-of-freedom) line-conjugate tooth surface couple, the meshing theory of the 1DOF point-conjugate tooth surface couple, the meshing theory based on reference point and the meshing theory of the 2DOF (two degree-of freedom) conjugate tooth surface couple. The meshing theory of the 1DOF line-conjugate tooth surface couple is the one of the most fundamental branches in meshing theory of gears. Line-contact gearing and cutting engagement based on generating method, both of which can be studies through it. In view of this, meshing theory of the 1DOF line-conjugate tooth surface couple is implicit in all the monographs on gear meshing theory.

In 1955, Litvin[3] proposed a method about rotational and translational 4 × 4 matrices. This method provides an easy way to study meshing theory of the 1DOF line-conjugate tooth surface couple and has been widely used in the research on meshing theory. Since the 1960s, Litvin[7–9] has written a series of works on meshing theory for gear drives. In it, the meshing theory of conjugate surfaces and geometry of gear meshing forms were illustrated. Moreover, it provides an important reference for theoreticians and technicians in the gear industry.

Starting from the 1970s, based on the differential method and moving frame, a systematic study on meshing theory for gear drives was carried out. This study was started by the research group on gear engagement led by Yan[10–13] at Department of Mathematics of NanKai University. There were several significant concepts clarified, along with the related equations, including the meshing limit line, the curvature interference limit line, the limit curvature of the conjugate surface, and the induced normal curvature.

The generalized Euler and Bertrand formulas were derived in 1985 by Wu and Luo[14]. As a result, it is possible to determine the curvature and the geodesic torsion along any direction at any location on tooth surfaces without relying on the principal directions and principal curvatures. Following decades, based on the above methods, monographs have emerged on systematic study of gear meshing theory. Recently, a series of formulas for lubrication characteristic parameters of line-conjugate tooth surface couples, were proposed by Zhao[15]. Developed a set of evaluation methods to measure the relative quality of lubrication characteristic in the entire conjugate zone. Furthermore, the induced geodesic torsion and the generalized Euler and Bertrand formulas were presented in more compact and clearer. Zhao et al.[16], Mu et al.[17], Zhu et al.[18], Huai and Zhao[19] subsequently focuses primarily on researching the line-contact meshing theory of a novel worm drive. He develops an approach to judge the existence of the solution of complex nonlinear equations during this process, and he then applies this method to precisely depict the meshing limit line, the curvature interference limit line, conjugate area, and instantaneous contact line of various drives within the shaft cross section of worm pairs. Lately, Zhao[20] authored "Engineering differential geometry of curves and surfaces". The main purpose was to provide a description for the knowledge of differential geometry involved in meshing theory. It is a essential reference for workers and technicians who are beginning to learn about meshing theory.

Theoretically, point-contact gearing is commonly used in manufacture. The development of 1DOF point-contact gear meshing is due to the study of spiral bevel gear and hypoid gear. In the early 1960s, by means of Wildhaber's research, Baxter[2] proposed the Tooth Contact Analysis (TCA) method which has been successfully applied to the study of spiral bevel gear and hypoid gear. Litvin[7], Litvin and Kin[21] (1968) proposed the approach of local synthesis which was combined with the TCA method to investigate the point-contact characteristics of different transmission forms and verified that the point-contact gearing pair has a lower sensitivity to errors. Up to now, the meshing theory of the 1DOF point-conjugate tooth surface couple has become mature. and formed a theoretical system with the TCA (Tooth Contact Analysis) and the approach of local synthesis as the core content. An improved local synthesis approach was enhanced to solve the mismatch problem of gear drives by Zhao et al.[22], Meng et al.[23,24] and Chi et al.[25,26]. Zhao et al.[22] also derived more accurate equations for the relative principal curvatures and the relative principal directions at the instantaneous contact points of the two conjugate tooth surfaces.

In terms of contact form, the meshing of the 2DOF conjugate tooth surface is a typical point-contact engagement. Due to the complicated process, the 2DOF theory has not been massively popularized and applied in actual manufacturing. In 1975, Litvin et al.[27] was first to proposed the 2DOF theory of gearing. Since then, Gao and Li[28], Zhao et al.[29] and Dong et al.[30] have made in-depth studies some key issue in the 2DOF theory. Based on the meshing theory of 1DOF gears, Dong[31,32] conducted a systematic study on the meshing theory of the 2DOF gear. Some formulas for convenient engineering application were obtained, for instance, the curvature interference limit line function, the induced normal curvature and the induced geodesic torsion. In 2008, Zhao et al.[29] specifies that the generating surface of the 2DOT engagement is divided into two section by the meshing limit line: meshing zone and useless zone, and gives the conditions for useless zone, and describes the meshing characteristics of the two zones. Besides, Zhao et al.[33] investigated the meshing principle for generating a cylindrical gear using an Archimedes hob with 2DOF, the root cutting properties of the workpiece, and the proximity between the tooth surface of the workpiece and the involute helical surface from curvature properties.

For the research of the meshing theory of a point conjugate gear pair, the key lies in the solution of the nonlinear contact system. Only the parameters at the contact point can be accurately obtained to analyse the meshing properties of the transmission pair. It is quite challenging to determine whether a solution exists for the complicated system. When the system has no solution, it is hard to figure out whether the initial value of system is irrational or the system itself doesn't have solution. If the initial value is irrational, whether the transmission pair's design parameters and process parameters are irrational, or the precision of the initial value is insufficient, these issues require in-depth investigation. In addition, it is necessary to investigate 3D visualization geometry modeling technologies for complex space surfaces. The tooth surface equation is primarily a complex implicit function form for a tooth surface with complex geometry. Correct computer simulation analysis and study on the contact strength and bearing capacity of the tooth surface is currently limited by the lack of an efficient geometric modeling technique that can accurately portray the 3D model of the pertinent tooth surface.

3. Conclusion and prospect

Gearing has been utilized in production for hundreds of years since its birth, and the meshing theory of gearing has been studied in depth and systematically by numerous researchers. The continued advancement of gearing technology has been fueled by the growth of meshing theory. Specifically, enhanced knowledge of gear meshing characteristics, ongoing advancements in gear design and manufacturing techniques, encouragement of the development of novel drive types, etc. It is expected that when new types of drives are developed and the manufacturing process is improved, the meshing theory of gearing will be used more frequently and continue to advance.

Conflicts of interest

The authors declare no conflict of interest.

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