

Article

Analysis of the effectiveness of object coordinates estimation in WSN using the RSS method

Vladimir Ivanovich Parfenov

Physical Department, Voronezh State University, Voronezh 394018, Russia; vip@phys.vsu.ru

CITATION

Parfenov VI. Analysis of the effectiveness of object coordinates estimation in WSN using the RSS method. *Computer and Telecommunication Engineering*. 2024; 2(1): 2400.
<https://doi.org/10.54517/cte.v2i1.2400>
0

ARTICLE INFO

Received: 27 November 2023
Accepted: 6 February 2024
Available online: 23 February 2024

COPYRIGHT

Copyright © 2024 by author(s).
Computer and Telecommunication Engineering is published by Asia Pacific Academy of Science Pte. Ltd. This work is licensed under the Creative Commons Attribution (CC BY) license.
<https://creativecommons.org/licenses/by/4.0/>

Abstract: The methods of estimating the coordinates of sensor nodes based on the measurements made at the “anchor” nodes are widely used in WSNs. In particular, such methods include the RSS method, which is based on measuring the power of signals coming from sensors. The article shows that a similar method can be used for estimating the coordinates of an observation object in the WSN. The efficiency of measuring the coordinates of such an object in the presence of power measurement errors is analyzed. The conditions for increasing this efficiency have been identified. It is shown that the estimation is biased, but the magnitude of the bias is practically independent of the observational conditions and, therefore, can be easily compensated.

Keywords: signal strength; trilateration; bias and dispersion of estimate

1. Introduction

Currently, due to the global trends of increasing energy costs, improving the efficiency and environmental friendliness of production, and ensuring the safety of human life and the environment, much attention is paid to the development and application of innovative technologies, including wireless sensor systems. A wireless sensor network (WSN) is a distributed, self-organizing system of multiple sensors designed to monitor physical phenomena or environmental conditions, as well as actuators interconnected by radio channels [1]. Due to their high flexibility, extended surveillance coverage, reliability, mobility, and cost-effectiveness, WSNs have wide application and high potential in the fields of military surveillance, security, and environmental monitoring [2,3]. In particular, a distributed security system can serve as an example of such a WSN. Decisions about the presence or absence of penetration into a protected object in such systems can be made simultaneously by several sensors according to a certain algorithm. These decisions are further transmitted via radio channel to the fusion center (FC), in which local decisions made in the form of binary information such as “yes” or “no” are combined and the final decision on the presence or absence of intrusion on the object is made in order to improve the system efficiency. It is obvious that for the effective functioning of the WSN, it is necessary to know as accurately as possible the mutual location of the sensor nodes and the object of observation (the target). Without this knowledge, the information coming from the sensor nodes is incomplete [4,5]. The following example can be given to illustrate what has been said: If the sensor network is designed to detect illegal entry into a protected area, the value of the received trespass information, without the coordinates of the trespasser, cannot be considered satisfactory. In addition, location information is also required for routing tasks, object trajectory tracking, etc. All this indicates the need to determine the location of both the sensor nodes themselves and the object of

observation. This task will be called the localization task, for short. This task involves a whole set of special methods that allow you to estimate coordinates based on some information from the surrounding space, in particular from other sensors. Of course, the use of GPS navigators would make it possible to solve such a problem quite easily. However, in the WSN, where significant limitations are imposed on the resources used, this approach is unpromising [6].

2. Known localization methods

Let us first consider the well-known localization methods used in WSNs [7]. It is known that the locations of sensor nodes can be found using either global or relative metrics [8]. A global metric is used when there is some global reference, as in GPS. At the same time, relative metrics are based on the use of some arbitrary coordinate systems rather than global ones. For example, the location of sensor nodes can be expressed in terms of distances to other sensors. Let's take into account that in addition to the ordinary sensors with unknown coordinates, the network may contain so-called "anchor" nodes whose coordinates are known. They are used to localize the other sensors. Let us consider some ways of solving the localization problem. Basically, these methods are based on measuring at each node some characteristics of the signals. The nodes then exchange these characteristics. These characteristics include propagation time, angle of arrival, signal strength, etc. Briefly, the essence of such methods is as follows. For example, the concept of one such method, called ToA (Time of Arrival) method [9], is based on the fact that the distance between the transmitter and the receiver can be found by measuring the propagation time of the signal. If t_1 and t_2 are the times of signal emission and reception (measured at the transmitter and receiver, respectively), and v is the speed of signal propagation, then the distance between these points is equal to $v(t_2 - t_1)$. A significant limitation of this method is that the measurement requires an accurate clock (to synchronize the transmitter and receiver).

The TDoA (Time Difference of Arrival) method [9–11] is based on using either multiple "anchor" nodes or two signals, propagating at different speeds. Indeed, in the first case, arrival times from several "anchor" nodes are measured, resulting in an estimate of not the absolute distance from the sensor to the "anchor" node (as in the ToA method), but the relative distance. The second approach of this method requires two different signals (e.g., a radio signal with velocity v_1 and an acoustic signal with velocity v_2) to be emitted one after another. If we measure the arrival time of the radio signal t_2 , as well as the arrival time of the acoustic signal t_4 emitted immediately one after another without delay, the distance between the sensors can be calculated by the formula $(v_1 - v_2)(t_4 - t_2)$. Another way to localize sensors is the AoA (Angle of Arrival) method [12,13], based on measuring the direction to the radiation source. However, this requires an antenna array (when using radio signals) or a microphone array (when using acoustic vibrations) to be located at the sensor node, which is absolutely impractical in a WSN. Finally, there is another localization method (RSS method – Received Signal Strength) [14–17]. It is based on the well-known fact that the power of the received signal in free space decreases inversely proportional to the square of the distance to the transmitter (if the channel model is more complex, for

example, it implements multipath propagation, reflection, etc., the quadratic law d^{-2} should be replaced on $d^{-\nu}$, where $\nu = 3..5$). The advantage of this method is that it does not require synchronization between different nodes. However, the accuracy of this method is significantly affected by external factors, especially if they change drastically in time.

Direct localization of sensors or any radiation object is based on the range characteristics measured by one of the methods listed above (ToA, TDoA, AoA, RSS). For this purpose, well-known methods of triangulation, trilateration, etc., are used [18]. In prospect, the problem of estimating the coordinates of an observation object in a WSN from the measured powers of the received signals in the sensor nodes will be considered.

3. Trilateration method for estimating the coordinates of a radiation object in WSN

As is known [19], the trilateration method is used when there are a number of “anchor” nodes with known coordinates and it is necessary to measure the coordinates of a node with unknown coordinates. In this case, the RSS method is used to measure the distances from the “anchor” nodes to the desired node. If a “flat” localization problem is considered, then three “anchor” nodes are sufficient to solve it. In this case, circles are drawn with centers where the “anchor” nodes are located and radii equal to the measured distances to the desired node. The desired coordinate is located somewhere in the vicinity of the intersection of these circles. Let us specify the conditions of the problem.

Let’s assume that we are solving the problem of estimating the coordinates of an object based on the signals received from it in the “anchor” sensor nodes. Suppose that there are K sensors in total; we denote a priory known coordinates by $(x_k; y_k)$, ($k = 1, \dots, K$). Let us also assume that the radiation object is located at a certain point with coordinates $(X_0; Y_0)$.

In each sensor, the power of the incoming signal P_k ($k = 1, \dots, K$) is measured, and this information is transmitted via radio channels to the fusion center (FC), where the final processing of this information is carried out. Again, for simplicity, we assume that all radio communication channels are ideal, i.e., this information is transmitted to the FC without distortion. Note that in the future it is expedient to consider also the case when such communication channels are not ideal. The number “ i ” of some sensor, closest to the object of observation (it is obvious that $P_i = \max(P_k), k = 1, \dots, K$) is determined in the fusion center based on the data received from the sensors.

Let us write an obvious system consisting of K equations:

$$\begin{cases} (x - x_1)^2 + (y - y_1)^2 = d_1^2, \\ \dots \\ (x - x_i)^2 + (y - y_i)^2 = d_i^2, \\ \dots \\ (x - x_K)^2 + (y - y_K)^2 = d_K^2. \end{cases}$$

Here d_k ($k = 1, \dots, K$)—distances from the k -th sensor node to the observation object. Let us subtract the i -th equation from each equation. As a result, the system of equations will already contain $K-1$ equations and have the form

$$\begin{cases} v_1 x + \chi_1 y = b_1, \\ \dots \\ v_K x + \chi_K y = b_K, \end{cases}$$

where $v_k = 2(x_k - x_i)$, $\chi_k = 2(y_k - y_i)$, $b_k = x_k^2 - x_i^2 + y_k^2 - y_i^2 + d_i^2 - d_k^2$. This system of equations can be rewritten in matrix form as

$$AX = b, \quad (1)$$

where $A = \begin{pmatrix} v_1 & \chi_1 \\ \dots & \dots \\ v_K & \chi_K \end{pmatrix}$ —matrix of size $(K - 1) \times 2$; $X = \begin{pmatrix} x \\ y \end{pmatrix}$ —vector of the size 2×1

of unknown object coordinates; $b = \begin{pmatrix} b_1 \\ \dots \\ b_K \end{pmatrix}$ —column vector of size $(K - 1) \times 1$.

Obviously, the system of equations (1) is overdetermined, in which the number of unknowns is less than the $K-1$. The solution of such a system can be found by the least squares method. In this case, the estimate of the object's coordinates is obtained in the form

$$\hat{X} = C \cdot b, \quad (2)$$

where $C = (A^T A)^{-1} A^T$ is a matrix of dimension $2 \times (K - 1)$.

Obviously, in (2) the vector C is known exactly, since the coordinates of all sensor nodes $(x_k; y_k)$ are known a priori. At the same time, the vector b includes differences of the form $d_i^2 - d_1^2, \dots, d_i^2 - d_K^2$, depending on d_k —the distance from the object to the k -th sensor node. These distances can be measured by RSS method, measuring the incoming signals strengths at each node.

4. Statistical characteristics of the coordinate of the observation object

In the absence of signal power measurement errors, the estimate (2) coincides with the true values of the object coordinates. However, any measurements are carried out with errors, in particular, due to the presence of measurement noise. Therefore, it is necessary to determine the degree of influence of these measurement errors on the accuracy of object coordinates estimation. Usually in the literature for this purpose, some random component is simply added to the measured power. However, neither the choice of the distribution itself nor the choice of its parameters is explained in any way [20]. Therefore, we will try to find explicit expressions for the statistical characteristics of the estimates of the coordinates of the radiation object \hat{X} (2), based on the known characteristics of the signal energy measurement by the energy receiver. For this purpose, we will sequentially find the statistical characteristics of the corresponding random variables. This will allow us to find the characteristic functions of the object coordinate estimates. On their basis, of course, the moments of any order of these estimates can be found, for example, the moments of the estimates of coordinates can be found.

Let us take into account that in each sensor node, the processing of the signal coming from the observed object is based on the use of an energy receiver [21]. In addition, let us consider that the amplitude of the signal arriving at the k -th sensor can be represented in the form $a_k = \sqrt{P_0} d_0 / d_k$, where P_0 is the power of the signal arriving at the reference point with a known distance to the target d_0 . Then, in

accordance with the study by Kostylev [22], the probability density of the signal at the output of the energy receiver in the k -th sensor node will be described by the expression of the form

$$p_{E_k}(x) = \frac{1}{2} \exp\left(-\frac{x+\lambda_k}{2}\right) \left(\frac{x}{\lambda_k}\right)^{\frac{n-1}{4}} I_{\frac{n-1}{2}}(\sqrt{\lambda_k x}), x \geq 0. \quad (3)$$

The probability density (3) is a non-central χ^2 -distribution with n degrees of freedom, depending on the signal base, and the non-centrality parameter λ_k coincides with the signal-to-noise ratio (SNR) $\lambda_k = z_k^2 = \frac{2a_k^2 T}{N_0} = qa_k^2 = qP_0 \left(\frac{d_0}{d_{0k}}\right)^2$, where N_0 —one-sided white noise power spectral density; T —duration of the observation interval; d_{0k} —true distance between the signal source and the k -th sensor; $q = 2a^2 T/N_0$; $I_n(\cdot)$ —modified n -th order Bessel function; a —unit amplitude.

Let us represent the power of the signal arriving at the k -th sensor as $P_k = \gamma E_k^2$. Consequently, the power probability density can be written, taken into account (3), as

$$p_{P_k}(x) = \frac{x^{\frac{n-3}{8}}}{4\lambda_k^{\frac{n-1}{4}} \gamma^{\frac{n+1}{4}}} \exp\left(-\frac{1}{2}\left(\sqrt{\frac{x}{\gamma}} + \lambda_k\right)\right) I_{\frac{n-1}{2}}\left(\sqrt{\lambda_k}\left(\frac{x}{\gamma}\right)^{1/4}\right), x \geq 0. \quad (4)$$

Next, we will find the distribution of the squared distance $R_k = d_k^2$ to the k -th sensor, taking into account that power and distance are related by the relation $P_k = P_0(d_0/d_k)^v$. Here the parameter v can take values from 2 to 5 depending on the conditions of wave propagation in the channel. As a result, taking into account (4) we have

$$p_{R_k}(x) = \frac{vP_0^{\frac{(n+1)}{8}} d_0^{v\frac{(n+1)}{8}}}{8\lambda_k^{\frac{n-1}{4}} \gamma^{\frac{n+1}{4}} x^{\frac{v(1+n)}{4}+1}} \exp\left(-\frac{1}{2}\left(\sqrt{\frac{P_0 d_0^{v/2}}{\gamma x^{v/4}}} + \lambda_k\right)\right) I_{\frac{n-1}{2}}\left(\sqrt{\lambda_k d_0^{v/2}}\left(\frac{P_0}{\gamma x^{v/2}}\right)^{1/4}\right), x \geq 0. \quad (5)$$

In the vector b (see formula (2)), the k -th element of this vector can be rewritten as $b_k = r_k + \Delta R_k$, where $r_k = x_k^2 - x_i^2 + y_k^2 - y_i^2$, $\Delta R_k = R_i - R_k$, $k = 1, \dots, K$, $k \neq i$. Taking into account the independence of the random variables R_i and R_k , we find the probability density of the variable ΔR_k : $p_{\Delta R_k}(x) = \int_0^\infty p_{R_k}(y)p_{R_i}(y+x)dy$. The probability density of the variable b_k will then be written in the form

$$p_{b_k}(x) = \int_0^\infty p_{R_k}(y)p_{R_i}(x-r_k+y)dy, \quad (6)$$

where should I substitute (5).

Let us rewrite the expressions for the estimates of the coordinates of the observation object $\hat{X} = \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix}$ on the basis of (2) in the form $\hat{x} = \sum_{k=1}^K c_{1k} b_k$, $\hat{y} = \sum_{k=1}^K c_{2k} b_k$. Taking into account formulas (5), (6), we can write the following expressions for characteristic functions of random variables \hat{x} and \hat{y} :

$$\begin{aligned} \theta_{\hat{x}}(u) &= \prod_{\substack{k=1 \\ k \neq i}}^K \exp(juc_{1k}r_k)\theta_{R_k}(-uc_{1k})\theta_{R_i}(uc_{1k}), \\ \theta_{\hat{y}}(u) &= \prod_{\substack{k=1 \\ k \neq i}}^K \exp(juc_{2k}r_k)\theta_{R_k}(-uc_{2k})\theta_{R_i}(uc_{2k}), \end{aligned} \quad (7)$$

where $\theta_{R_k}(u)$ is the characteristic function of the random variable R_k , which can be found from (5) taking into account the known relationship between the characteristic function and the probability density [23].

5. Numerically results and discussion

To quantify the efficiency of measuring the coordinates of the radiation object, we will use the conditional biases and dispersions of these estimates:

$$\begin{aligned} d(\hat{x}|X_0) &= \langle \hat{x} - X_0 | X_0 \rangle, D(\hat{x}|X_0) = \langle (x - \langle x \rangle)^2 | X_0 \rangle, \\ d(\hat{y}|Y_0) &= \langle \hat{y} - Y_0 | Y_0 \rangle, D(\hat{y}|Y_0) = \langle (y - \langle y \rangle)^2 | Y_0 \rangle. \end{aligned}$$

Here, angle brackets denote the operation of averaging over the ensemble of realizations, $(X_0; Y_0)$ – the true values of the object coordinates. These characteristics can be easily found taking into account (7), since, knowing the characteristic function of a random variable, the initial moments of arbitrary m -th order can be found by the formulas [23]

$$m_{\hat{x}_m} = \frac{1}{j^m} \left[\frac{\partial^m \theta_{\hat{x}}(u)}{\partial u^m} \right]_{u=0}, m_{\hat{y}_m} = \frac{1}{j^m} \left[\frac{\partial^m \theta_{\hat{y}}(u)}{\partial u^m} \right]_{u=0}.$$

The calculation according to these formulas was performed numerically taking into account expressions (5)–(7). For certainty the following was assumed. The number of sensors was chosen to be 5. The coordinates of sensor nodes were set equal to: $(0; -0,213); (0,25; 0,13); (0,5; -0,31); (0,75; -0,157); (1; 0,0456)$. In addition, the following was assumed in the calculations: $P_0 = 1, d_0 = 1, \gamma = 1, \nu = 2$. The coordinates of the object of observation were chosen equal to $(X_0; Y_0) = (0,393; 0,51)$. The values n, q and K were varied. **Figure 1** displays the dispersion of object coordinate estimates, find in such conditions.

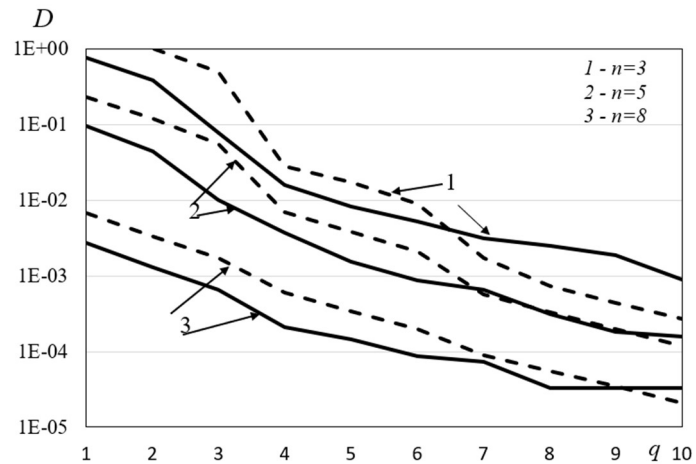


Figure 1. Dispersion of object coordinate estimates for a different number of degrees of freedom n .

In **Figure 1**, the solid lines show the dependences of the dispersions of the coordinate's estimates \hat{x} , and the dashed lines show the dependences of the dispersions of the coordinates \hat{y} on the SNR q . The results of calculations indicate the following. The coordinates estimations have some bias, weakly depending on the parameters n , q and K . Taking into account the weak dependence of the estimation bias on the observational conditions, we can state that the estimation according to (2) has a systematic error. However, such an error can be easily compensated, for example [24]. In order to compensate the bias level, first the positioning system should figure out how transmitting power of each node is biased relatively. Therefore, before the positioning procedure, the positioning system should collect several RSS measurements from each node. For this purpose, we calculate the average power value m_k in each sensor node ($k = \overline{1, \dots, K}$) based on the results of several measurements. Then we find an estimate of the bias level using the formula

$$\hat{B}_k = m_k - \frac{1}{K} \sum_{k=1}^K m_k. \quad (8)$$

In Equation (8), \hat{B}_k is estimate of the bias level, where K is number of nodes in system. That is, the positioning system can estimate \hat{B}_k by subtracting average of m_k from collected RSS average of certain node P_k . Since the average of m_k can be a criterion about the bias level of certain node, it is appropriate that the system estimate the relative bias level in this way

$$\hat{P}_k = P_k - \hat{B}_k. \quad (9)$$

In (9), P_k is received biased RSS, \hat{B}_k is estimate of bias level, and \hat{P}_k is compensated RSS; in other words, estimate of unbiased RSS. Finally, the system has compensated RSS, then these values are used to calculating the coordinates of estimated position by trilateration technique, previously described. No matter how serious a bias problem is, the system can almost remove the influence of the bias problem if it correctly estimate the bias level [14].

The conditional estimation dispersion, as evidenced by the analysis of **Figure 1**, decreases significantly with increasing both the parameter n , characterizing the signal base, and the parameter q , characterizing the signal-to-noise ratio (SNR) in the object-sensor channel. At the same time, the increase in the number of sensors not lead to such a significant decrease in the dispersion of the coordinate estimation as expected.

6. Conclusion

In spatially distributed wireless sensor networks, for the correct solution of complex detection tasks, estimation of radiation parameters, classification of objects of observation, etc., it is necessary to know the coordinates of both sensor nodes themselves and the object of observation. The paper shows how to find the coordinates of the object using the trilateration method and the RSS method when the coordinates of sensor nodes are known. The estimation of the object coordinates was done using the well-known least squares method. In addition, the accuracy of this estimation, as described by the conditional bias and dispersion, was determined. It was shown that the efficiency of coordinate estimation carried out by this method depends on the parameters of the signal emitted by the object. In particular, the estimation efficiency

increases if the parameters n and q increase. Further research in this context could consist of the following. In particular, it is reasonable to investigate the influence on the efficiency of object coordinate estimation of interference in the “sensor-FC” channels, as it is done [25].

Conflict of interest: The authors declare no conflict of interest.

References

1. Sharma R. Advanced wireless sensor networks. In: Chapter in book Emerging computing paradigms: Principles, advanced and applications. Wiley Data and Cybersecurity; 2022. pp. 177-191.
2. Kwasinski A, Chande V. Recent applications and emerging designs in source-channel coding. In: Chapter in book Joint source-channel coding. Wiley-IEEE Press; 2022. pp. 335-379.
3. Khanna A, Kaur S. Internet of Things (IoT), Applications and Challenges: A Comprehensive Review. Wireless Personal Communications. 2020; 114(2): 1687-1762. doi: 10.1007/s11277-020-07446-4
4. Liu Y, Yang Z. Location, Localization, and Localizability. Springer New York; 2011. doi: 10.1007/978-1-4419-7371-9
5. Zekavat SAR. Handbook of Position Location: Theory, Practice, and Advances. IEEE Press-Wiley; 2011.
6. Faragher R, Harle R. Location Fingerprinting with Bluetooth Low Energy Beacons. IEEE Journal on Selected Areas in Communications. 2015; 33(11): 2418-2428. doi: 10.1109/jsac.2015.2430281
7. Win MZ, Shen Y, Dai W. A Theoretical Foundation of Network Localization and Navigation. Proceedings of the IEEE. 2018; 106(7): 1136-1165. doi: 10.1109/jproc.2018.2844553
8. Zhai D, Lin Z. RSS-based indoor positioning with biased estimator and local geographical factor. 2015 22nd International Conference on Telecommunications (ICT). Published online April 2015. doi: 10.1109/ict.2015.7124718
9. Lin M, Wang W, Liu C. Preventing Hostile TOA/TDOA Localization with AF Relay. IEEE Communications Letters. 2023; 27(4): 1085-1089. doi: 10.1109/lcomm.2023.3247871
10. Wang Y, Ho KC. TDOA Positioning Irrespective of Source Range. IEEE Transactions on Signal Processing. 2017; 65(6): 1447-1460. doi: 10.1109/tsp.2016.2630030
11. Wu P, Su S, Zuo Z, et al. Time Difference of Arrival (TDoA) Localization Combining Weighted Least Squares and Firefly Algorithm. Sensors. 2019; 19(11): 2554. doi: 10.3390/s19112554
12. Zuo P, Peng T, Wu H, et al. Directional source localization based on RSS-AOA combined measurements. China Communications. 2020; 17(11): 181-193. doi: 10.23919/jcc.2020.11.015
13. Nguyen NH, Dogancay K, Kuruoglu EE. An Iteratively Reweighted Instrumental-Variable Estimator for Robust 3-D AOA Localization in Impulsive Noise. IEEE Transactions on Signal Processing. 2019; 67(18): 4795-4808. doi: 10.1109/tsp.2019.2931210
14. Li X. RSS-Based Location Estimation with Unknown Pathloss Model. IEEE Transactions on Wireless Communications. 2006; 5(12): 3626-3633. doi: 10.1109/twc.2006.256985
15. Niu R, Vempaty A, Varshney PK. Received-Signal-Strength-Based Localization in Wireless Sensor Networks. Proceedings of the IEEE. 2018; 106(7): 1166-1182. doi: 10.1109/jproc.2018.2828858
16. Peng T, Zuo P, You K, et al. Bounds and Methods for Multiple Directional Sources Localization Based on RSS Measurements. IEEE Access. 2019; 7: 131395-131406. doi: 10.1109/access.2019.2940650
17. Pflaum F, Erhardt S, Weigel R, et al. RSSI-based localization with minimal infrastructure using multivariate statistic techniques. In Proc. of IEEE Topical Conference on Wireless Sensors and Sensor Networks; 2017: 69-72. doi: 10.1109/ECAI.2018.8679093
18. Gezici S. A Survey on Wireless Position Estimation. Wireless Personal Communications. 2007; 44(3): 263-282. doi: 10.1007/s11277-007-9375-z
19. Dargie W, Poellabauer C. Fundamentals of wireless sensor networks: Theory and practice. Wiley & Sons; 2010.
20. Hernandez-Perez E, Navarro-Mesa JL, Martin-Gonzales S, et al. Path loss factor estimation for RSS-based localization algorithms with wireless sensor networks. In: 19th European Signal Processing Conference (EUSIPCO 2011). Barcelona, Spain; 2011. pp. 1994-1998.

21. Babu RS, Suganthi M. Review of energy detection for spectrum sensing in various channels and its performance for cognitive radio applications. *American Journal of Engineering and Applied Sciences*. 2012; 5(2): 151-156. doi: 10.3844/ajeassp.2012.151.156
22. Kostylev VI. Energy detection of a signal with random amplitude. In: *IEEE International Conference on Communications. Conference Proceedings*; 2002. pp. 1606-1610.
23. Bhat BR. *Modern Probability Theory*. New Academic Science; 2019.
24. Choi H., Jin H., Kim S.C. RSS bias compensation in BLE beacon based positioning system. In: *2017 Ninth International Conference on Ubiquitous and Future Networks (ICUFN)*.
25. Parfenov VI, Le VD. Optimal fusion rule for distributed detection with channel errors taking into account sensors' unreliability probability when protecting coastlines. *International Journal of Sensor Networks*. 2022; 38(2): 71-84. doi: 10.1504/IJSNET.2022.121157