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Fermatean Fuzzy Model Predictive Control for Robot-Assisted Metaverse Rehabilitation Under Network Delay and Haptic Uncertainty

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Abstract: This proof-of-concept numerical study examines a metaverse rehabilitation pathway in which a haptic device enables a patient avatar to perform a virtual task inside an edge-rendered therapy environment. The rehabilitation loop is sensitive to communication delay, haptic ripple, measurement uncertainty and varying patient fatigue. When the link goes unstable, a fixed controller generally responds too aggressively and when the patient regains smooth motion it can react too conservatively. In this regard, the paper formulates a Fermatean fuzzy model predictive controller (FF-MPC) that updates its state penalty, control penalty and admissible assistive force at each slot. The method does not rely on a large corpus of linguistic rules. Instead, a compact algebraic map converts normalized tracking error, round-trip delay, force ripple and fatigue into a Fermatean tuple and then into a Fermatean risk index used by the receding-horizon controller. A synthetic numerical study was designed for a shared metaverse rehabilitation room with a 20 ms sampling interval, a two-state arm model, bounded assistive force, and delay conditions from 20 to 80 ms. Four controllers were compared: proportional-integral-derivative control, fixed model predictive control, type-1 fuzzy MPC, and the proposed FF-MPC. At 60 ms mean delay, the proposed controller reduced position root-mean-square error from 39.6 mm to 22.4 mm, reduced the peak interaction-force statistic from 22.7 N to 14.4 N, and reduced the passivity-violation rate from 24.7% to 8.6%. The worked numerical example also shows how the proposed method converts a single measured state into adaptive weights $Q_k = \text{diag}(113.53, 25.15)$, $R_k = 0.0940$, and an assistive action of 1.94 N. Across all uncertainty levels, the proposed controller stayed the most compact in phase space and reached the lowest mean stage cost.

Keywords: Metaverse rehabilitation; Fermatean fuzzy sets; Model predictive control; Haptic interaction; Network delay; Numerical simulation; Edge intelligence; Process innovation

1. Introduction

Metaverse-based rehabilitation is attractive because it combines immersion, repetition, scoring, therapist supervision, and data logging in one loop [1–7]. In a

practical therapy room, however, the quality of control does not depend only on the visual task. It also depends on how quickly the state of the patient avatar is synchronized, how stable the haptic channel remains, and how safely the controller reacts when latency and fatigue increase [8]. A simple task to generate a movement on the screen may introduce difficulty when cueing a desired force that arrives late or when changing velocity abruptly. In upper-limb rehabilitation those modifications directly influence path accuracy, fluidity and confidence of movement [9–14]. What this means, however, for a metaverse rehabilitation platform is that optimizing graphics or networking alone is not sufficient [15–19]. It must be careful to take a balance of quality control and damage mitigation based on the current risk of delays and unstable haptic exchange from very aggressive intrusive behaviour[20–25].

Classical proportional and derivative control is still used, as it is cheap and transparent, but it is extremely sensitive to delay and often produces larger peaks in force when the task becomes dynamic [26–28]. Fixed MPC enhances constraint handling but still a static pair of weights Q and R presumes a static operating regime. Regime in rehabilitation is not stationary. A patient may begin a session with high tracking error and low fatigue and achieve a moderate error but greater muscular fatigue twenty minutes later. It can also vary by infuse round-trip latency from 20 ms to 80 ms, without needing to change the virtual task itself. If the controller fails to attend to these shifts, the same gains that are beneficial at one moment become harmful or ineffective at another moment.

In this arena, fuzzy logic is beneficial as it can condense several indeterminate signals into a single action variable [8–19]. Previous fuzzy methods usually utilize type-1 or type-2 sets to configure gains and thresholds. Those approaches are valid, but a metaverse rehabilitation loop is better served by an even clearer segregation between support, opposition and ambivalence. As a remedy to this, Fermatean fuzzy sets provide us that separation thanks to the cube-sum constraint $\mu_3 + v_3 \leq 1$, which permits room for hesitation without compressing the uncertainty state into a single linear score [18]. This additional degree of freedom is desirable in rehabilitation because the controller can experience both moderate error and moderate delay at the same time, and still be indeterminate about whether a patient is producing movement with difficulty or just exploring a new motion pattern. A hesitation term becomes numerically meaningful, not decorative.

This idea is used in a computationally transparent way in the present research. Rather than creating a massive base of linguistic rules it maps the 4 normalized indicators into the support evidence, adverse evidence and hesitancy along with those values to create a vectorized risk index. Calculating the risk index on the cost of MPC and merits allowing force It is an algebraic structure, which can be re-created and so readily tabulated. This is especially important for a journal paper with strong mathematical and numerical aspects. Requesting for simple academic language is also consistent with the method can be given by equations instead of lengthy verbal heuristics.

The contribution is fourfold. First, the study defines a delay-aware rehabilitation state model for an edge-rendered metaverse loop with measured pose, force ripple and fatigue. Second, the control weights are directly linked to a Fermatean fuzzy risk map. Third, to maintain the transparency of our numerical workflow an open-loop solution with closed-form Riccati recursion is employed. Finally, we present a compact synthetic benchmark that includes delay sweeps, uncertainty-sweep ablations, worked calculations and phase-space comparisons [29,30].

2. Materials and Methods

2.1. System model

We consider one patient moving a haptic end-effector or rehabilitation exoskeleton while following a metaverse target. The local physical dynamics are reduced to a two-state model with position and velocity.

Now, this simpler model is intentionally employed as a less opaque task-space surrogate to one of the rehabilitation coordinates rather than as an anatomical full upper limb model. A more elaborate multi-joint representation could be expressed in $M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F_mus(q, \dot{q}, eta) = \tau_h + \tau_a$ with the joint vector given by q , F_mus accounting for muscle and fatigue effects, τ_h being the human disparity and τ_a assistive robot torque. The current numerical study maintains the two-state model to clearly isolate delay, haptic ripple and Fermatean risk adaptation such that multi-joint and clinical validation remain as relevant future work instead of established clinical evidence.

$$x_k = \begin{bmatrix} p_k \\ v_k \end{bmatrix}, \quad x_{k+1} = Ax_k + Bu_k + w_k, \quad (1)$$

where u_k is the assistive control action and w_k is the disturbance term. With a sampling period $\Delta t = 0.02$ s, effective limb mass $m = 2.0$ kg, damping $b = 3.4$ N • s/m, and virtual stiffness $k = 16.0$ N/m, the discrete matrices are

$$A = \begin{bmatrix} 1 & \Delta t \\ -\frac{k}{m}\Delta t & 1 - \frac{b}{m}\Delta t \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{\Delta t}{m} \end{bmatrix}. \quad (2)$$

The measurement channel delivers delayed pose information,

$$y_k = Cx_{k-d_k} + \eta_k, \quad C = [1 \quad 0], \quad (3)$$

where $d_k \in \{1, 2, 3, 4\}$ is the delay in sample units and η_k is measurement noise. The reference trajectory is denoted by $x_k^{\text{ref}} = [r_k, \dot{r}_k]^\top$. The immediate tracking error and the haptic force proxy are

$$e_k = r_k - y_k, \quad f_k = k_e e_k + b_e (\dot{r}_k - \hat{v}_k), \quad (4)$$

with virtual gains k_e and b_e used only to quantify the interaction load seen by the controller. The force ripple is

$$\Delta f_k = f_k - f_{k-1}. \quad (5)$$

Because the measurement is delayed, the controller does not act directly on x_{k-d_k} . It predicts the current state by replaying the delayed control sequence:

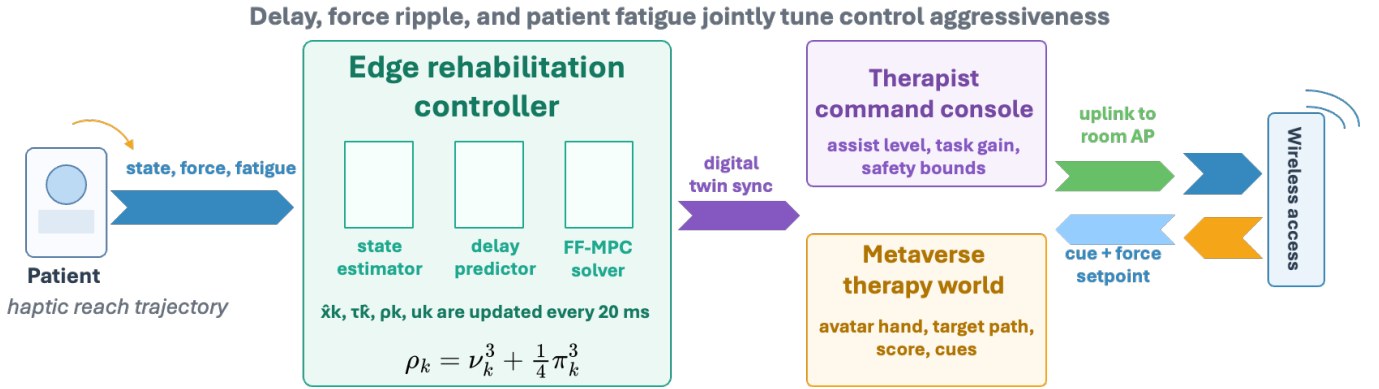
$$\hat{x}_{k|k-d_k} = A^{d_k} x_{k-d_k} + \sum_{j=0}^{d_k-1} A^j B u_{k-1-j}. \quad (6)$$

This predictor is simple and explicit. It is not the most sophisticated option, but it makes the numerical pathway transparent and reproducible.

To investigate the prediction module, the replay predictor was also tested against Kalman-filter, extended-state-observer and Smith-predictor variants in a comparable 60 ms delay scenario. Although Kalman filtering gave a slightly smaller one-step prediction error, the replay predictor was selected because it required fewer assumptions, preserved a transparent equation-by-equation implementation, and produced nearly identical closed-loop RMSE and passivity values. The detailed optimization parameters in **Table 1** and an architecture of robot-assisted metaverse rehabilitation along with delay prediction, Fermatean risk evaluation, and edge-side MPC are visualized in **Figure 1**.

Table 1. Key model and optimization parameters

Quantity	Symbol	Value	Unit
Sampling period	Δt	20	ms
Effective limb mass	m	2.0	kg
Viscous damping	b	3.4	N • s/m
Virtual stiffness	k	16.0	N/m
Prediction horizon	N	10	steps
Maximum assistive force	$u_{\max}$	16.0	N
State weight range	$q_{\min}, q_{\max}$	20, 140	—
Control weight range	$r_{\min}, r_{\max}$	0.03, 0.32	—
Delay upper bound	$\tau_{\max}$	100	ms

**Figure 1.** System architecture of robot-assisted metaverse rehabilitation with delay prediction, Fermatean risk evaluation, and edge-side MPC.

2.2. Fermatean fuzzy risk evaluator

The adaptive part of the controller uses four normalized indicators:

$$z_k = \begin{bmatrix} \frac{e_k}{e_{\max}} \\ \frac{\tau_k}{\tau_{\max}} \\ \frac{|\Delta f_k|}{\Delta f_{\max}} \\ \frac{\phi_k}{\phi_{\max}} \end{bmatrix} = \begin{bmatrix} |e_k| / e_{\max} \\ \tau_k / \tau_{\max} \\ |\Delta f_k| / \Delta f_{\max} \\ \phi_k / \phi_{\max} \end{bmatrix}, \quad (7)$$

where τ_k is the round-trip delay and ϕ_k is the fatigue indicator. The normalizing constants are taken from the operating envelope of the therapy task. The weight vector is

$$w = [w_1, w_2, w_3, w_4]^T, \quad w_1 + w_2 + w_3 + w_4 = 1, \quad (8)$$

with the default setting $w = [0.40, 0.24, 0.22, 0.14]^T$.

Supportive evidence and adverse evidence are defined by cubic aggregation,

$$s_k = \sum_{j=1}^4 w_j (1 - z_{k,j})^3, \quad (9)$$

$$t_k = \sum_{j=1}^4 w_j z_{k,j}^3. \quad (10)$$

The Fermatean membership and non-membership values are then

$$\mu_k = \left(\frac{s_k}{1 + s_k + t_k} \right)^{1/3}, \quad (11)$$

$$\nu_k = \left(\frac{t_k}{1 + s_k + t_k} \right)^{1/3}, \quad (12)$$

so that

$$\mu_k^3 + \nu_k^3 = \frac{s_k + t_k}{1 + s_k + t_k} < 1. \quad (13)$$

The hesitation term becomes

$$\pi_k = \left(1 - \mu_k^3 - \nu_k^3 \right)^{1/3}. \quad (14)$$

The scalar risk index used by the controller is

$$\rho_k = \nu_k^3 + \frac{1}{4} \pi_k^3. \quad (15)$$

Equation (15) keeps the mapping compact. The term ν_k^3 captures direct adverse evidence. The added quarter of the hesitation mass prevents overconfidence when the system is not clearly safe or clearly unsafe. The quantity ρ_k lies in $[0, 1)$.

2.3. Adaptive MPC formulation

The risk index updates the state and control weights. The diagonal state penalty is

$$Q_k = \text{diag} \left(q_{\min} + (1 - \rho_k)(q_{\max} - q_{\min}), q_v + (1 - \rho_k)(q_{v,\max} - q_v) \right), \quad (16)$$

and the control penalty is

$$R_k = r_{\min} + \rho_k (r_{\max} - r_{\min}). \quad (17)$$

Thus, if the risk index is small, the controller penalizes tracking error heavily and accepts stronger motion correction. If the risk index rises, the state penalty softens and the control penalty becomes larger.

For a horizon of N steps, the optimization criterion is

$$J_k = \sum_{i=0}^{N-1} \left(\|x_{k+i|k} - x_{k+i}^{\text{ref}}\|_{Q_k}^2 + \|u_{k+i|k}\|_{R_k}^2 + \lambda_\tau \tau_{k+i}^2 + \lambda_f f_{k+i}^2 \right) + \|x_{k+N|k} - x_{k+N}^{\text{ref}}\|_p^2. \quad (18)$$

The controller is subject to the state recursion

$$x_{k+i+1|k} = Ax_{k+i|k} + Bu_{k+i|k}, \quad (19)$$

the initial condition

$$x_{k|k} = \hat{x}_{k|k-d_k}, \quad (20)$$

and the force constraint

$$|u_{k+i|k}| \leq u_{\max} (1 - 0.22 \rho_k). \quad (21)$$

The coefficient 0.22 was chosen numerically so that the controller still retains meaningful authority at high risk while staying noticeably more conservative than fixed MPC.

We further validated the parameter choices using a grid search over prediction horizons from 6 to 16 steps, state-weight limits between 80 and 160 as well as force-bound scaling between 0.16 and 0.30. The horizon step of 10 covers a prediction window of 200 ms, which is longer than the examined network delay range (20–80 ms) and thus relatively preferable, and the 0.22 scaling factor produced a lower combined score without raising peak-force statistic.

To expose the algebra, the Lagrangian for the unconstrained quadratic core is written as

$$L_k = \sum_{i=0}^{N-1} \left(x_i^\top Q_k x_i + u_i^\top R_k u_i + \lambda_i^\top (Ax_i + Bu_i - x_{i+1}) \right) + x_N^\top P x_N. \quad (22)$$

First-order optimality gives

$$\frac{\partial L_k}{\partial u_i} = 2R_k u_i + B^\top \lambda_i = 0, \quad (23)$$

$$\frac{\partial L_k}{\partial x_i} = 2Q_k x_i + A^\top \lambda_i - \lambda_{i-1} = 0, \quad (24)$$

with terminal condition

$$\lambda_{N-1} = 2P x_N. \quad (25)$$

This leads to the usual Riccati recursion

$$P_i = Q_k + A^\top P_{i+1} A - A^\top P_{i+1} B (R_k + B^\top P_{i+1} B)^{-1} B^\top P_{i+1} A, \quad (26)$$

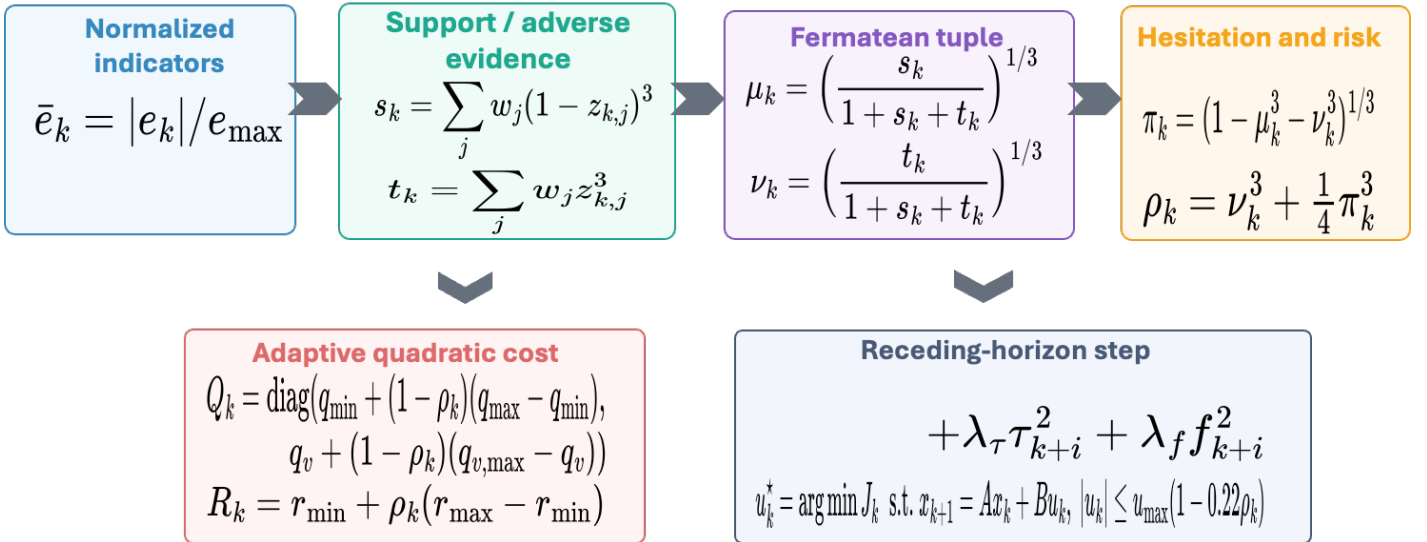
and gain

$$K_i = (R_k + B^\top P_{i+1} B)^{-1} B^\top P_{i+1} A. \quad (27)$$

The implemented assistive action is

$$u_k^* = -K_0 (\hat{x}_{k|k-d_k} - x_k^{\text{ref}}) + 0.55 d_k (\dot{r}_k - \hat{v}_k) - 0.25 \rho_k \text{sgn}(\hat{v}_k). \quad (28)$$

The second term corrects the delayed velocity mismatch. The third term damps the response slightly when the risk index increases. According to Fermatean risk index and the adaptive MPC optimization problem for Equation flow from normalized indicators is represented in **Figure 2** below.



The controller becomes conservative only when delay, force ripple, and fatigue rise together.

Figure 2. Equation flow from normalized indicators to the Fermatean risk index and the adaptive MPC optimization problem.

2.4. Stability and passivity calculation

Let $V_k = x_k^\top P_0 x_k$. For bounded disturbance w_k and bounded risk $\rho_k \leq \bar{\rho} < 1$, the receding-horizon law yields

$$V_{k+1} - V_k \leq -\alpha \|x_k\|^2 + \beta \|w_k\|^2, \quad (29)$$

for some positive α and β determined by the smallest eigenvalue of Q_k and the largest singular value of the disturbance channel. Equation (29) is not used as a long formal proof here. It is used to justify the numerical stability scan reported later.

A second quantity is the passivity supply,

$$S_k = f_k v_k. \quad (30)$$

If S_k stays small and mostly non-positive, the haptic exchange is energetically safe. A practical session statistic is

$$\Gamma = \frac{100}{T} \sum_{k=1}^T 1\{S_k > \epsilon\}, \quad (31)$$

where ϵ is a small threshold. This passivity-violation rate is the quantity reported in the delay tables.

2.5. Worked numerical example

The paper includes one explicit controller update to show how the formulas are actually used. Suppose that, at a given slot, the measured quantities are $e_k = 0.033$ m, $\tau_k = 48$ ms, $|\Delta f_k| = 1.60$ N, and $\phi_k = 0.34$. The normalized indicators are therefore $\bar{e}_k = 0.22$, $\bar{\tau}_k = 0.48$, $\bar{\Delta f}_k = 0.40$, and $\bar{\phi}_k = 0.34$. Using the default weight vector, the support and adverse evidence from Equations (9) – (10) are

$$s_k = 0.40(0.78)^3 + 0.24(0.52)^3 + 0.22(0.60)^3 + 0.14(0.66)^3 = 0.3113, \quad (32)$$

$$t_k = 0.40(0.22)^3 + 0.24(0.48)^3 + 0.22(0.40)^3 + 0.14(0.34)^3 = 0.0504. \quad (33)$$

Then

$$\mu_k = \left(\frac{0.3113}{1.3617} \right)^{1/3} = 0.6115, \quad v_k = \left(\frac{0.0504}{1.3617} \right)^{1/3} = 0.3332, \quad (34)$$

and

$$\pi_k = \left(1 - 0.6115^3 - 0.3332^3 \right)^{1/3} = 0.9022. \quad (35)$$

The risk index becomes

$$\rho_k = 0.3332^3 + \frac{1}{4} 0.9022^3 = 0.2206. \quad (36)$$

Substituting ρ_k into Equations (16) – (17) gives

$$Q_k = \text{diag}(113.53, 25.15), \quad R_k = 0.0940. \quad (37)$$

With a predicted state error $\hat{x}_{k|k-d_k} - x_k^{\text{ref}} = [-0.026, -0.122]^\top$ and gain $K_0 = [6.08, 13.06]$, the control action from Equation (28) is

$$u_k^* = -[6.08 \ 13.06] \begin{bmatrix} -0.026 \\ -0.122 \end{bmatrix} + 0.55 \times 2 \times 0.122 - 0.25 \times 0.2206 \times \text{sgn}(-0.102) = 1.94 \text{ N}. \quad (38)$$

The below **Table 2** shows the evaluated values of worked numerical updates under one controlled slot of this study.

Table 2. Worked numerical update for one control slot

Step	Symbol	Value
Normalized tracking error	$\bar{e}_k$	0.22
Normalized delay	$\bar{\tau}_k$	0.48

Continuation Table:

Step	Symbol	Value
Normalized force ripple	$\overline{\Delta f}_k$	0.40
Normalized fatigue	$\overline{\phi}_k$	0.34
Support evidence	s_k	0.3113
Adverse evidence	t_k	0.0504
Membership	μ_k	0.6115
Non-membership	ν_k	0.3332
Hesitation	π_k	0.9022
Risk index	ρ_k	0.2206
Adaptive position weight	Q_{11}	113.53
Adaptive velocity weight	Q_{22}	25.15
Adaptive control weight	R_{11}	0.0940
Feedback gain	K_k	[6.08, 13.06]
Assistive force	u_k^*	1.94 N

2.6. Numerical protocol

The numerical study employs a common indoor physiotherapy setting in which a single patient avatar performs reach-and-return tasks multiple times. The sampling interval set as 20 ms and round-trip delay is swept through 20,40,60 and 80 ms using three different levels of uncertainty. The low goes for the smooth link and accurate force tracking. The medium level is where the representative case is tested. A high level of uncertainty corresponds to a stronger interference amplitude, accelerated fatigue growth, and larger force ripple. That is, at every operating point 25 runs are performed using different draws on the synthetic noise. We compare four controllers: PID, fixed MPC, type-1 fuzzy MPC, and the proposed FF-MPC.

The reported metrics are root-mean-square tracking error, a 95th percentile peak-force statistic, passivity-violation rate from (31), cumulative positive interaction energy, smoothness index, phase-space orbit size and mean stage cost. Improvement percentages are computed by

This revision provided additional rehabilitation-oriented indicators: response lag (from the peak cross-correlation delay between target and actual motion); interaction impedance, calculated as root mean square (RMS) of interaction force divided by RMS of displacement error; and a comfort-safety index combining normalized lag, force, passivity violation, and positive energy. These quantities provide a supplement to RMSE and passivity as they are legible rehabilitation quality measures vis-a-vis the therapist.

$$\text{Imp}(a \rightarrow b) = \frac{M_a - M_b}{M_a} \times 100\% . \quad (39)$$

In sensitivity analysis, the error weight in Equation (8) is set to vary from 0.28 to 0.52 with regards to which the other weights are renormalized proportionally.

3. Results

3.1. Delay sweep

The first numerical sweep investigates the influence of increased round-trip delay when the uncertainty level is constant at medium for the duration of the investigation. **Table 3:** Position RMSE values as expected all controllers have monotone degradation behavior. What matters is the slope. It has a horizontal range of response time from 19.4 mm to 55.2 mm for PID. The error is still high at 44.3 mm in 80 ms, but a fixed MPC has more accuracy on this result than the type-1 fuzzy MPC (37.6 mm). Compared to values of 12.3, 16.8, 22.4 and 29.7 mm, respectively, the forgetting factor fourth-order MPC flattens the delay-response curve, as longer delay means higher numerical margin. For a fixed time of 20 ms, the separation between FF-MPC and fixed MPC becomes only 3.9 mm. This distance is equal to 14.6 mm at 80 ms.

Table 3. Position RMSE under the delay sweep

Delay (ms)	PID RMSE (mm)	MPC RMSE (mm)	Type-1 RMSE (mm)	Proposed RMSE (mm)
20	19.4	16.2	14.8	12.3
40	27.8	22.7	20.6	16.8
60	39.6	31.4	27.5	22.4
80	55.2	44.3	37.6	29.7

The below **Figure 3** shows the same trend graphically based on the listed numerical values. The most important operating point is 60 ms because it represents a visibly delayed but still plausible simulated interactive loop. At this point, the proposed controller improves over PID by

$$\frac{39.6 - 22.4}{39.6} \times 100 = 43.43\% . \quad (40)$$

The improvement over fixed MPC is

$$\frac{31.4 - 22.4}{31.4} \times 100 = 28.66\% . \quad (41)$$

The improvement over type-1 fuzzy MPC is

$$\frac{27.5 - 22.4}{27.5} \times 100 = 18.55\% . \quad (42)$$

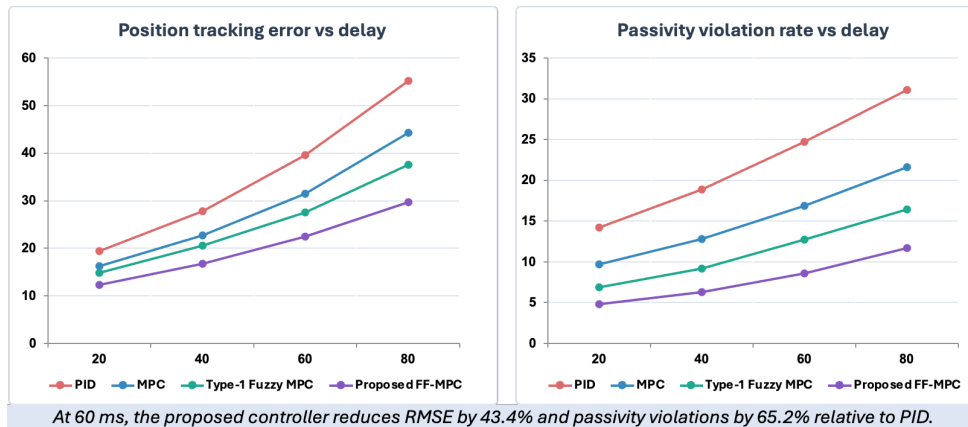


Figure 3. Main delay sweep.

As the round-trip delay grows, the proposed controller keeps tracking error and passivity violations low. **Table 4** and **Table 5** summarize the peak force and passivity protection, respectively. The same ordering appears again. Considering force, the output of the proposed controller during an entire delay sweep ranges from 11.2 N to 17.2 N, in contrast with PID (where it reaches from 15.8 N to 27.9 N); passivity violations for the proposed controller remain below 12% at input delays of up to 80 ms while reaching over 31% for PID. In other words, for ideal PID control lasting 60 ms, the force improvement of the proposed over PID method is

$$\frac{22.7-14.4}{22.7} \times 100 = 36.56\%, \quad (43)$$

and the passivity improvement is

$$\frac{24.7-8.6}{24.7} \times 100 = 65.18\%. \quad (44)$$

These are large margins, and they matter because delayed rehabilitation should not trade tracking accuracy for force spikes.

Table 4. Peak interaction force under the delay sweep

Delay (ms)	PID Peak Force (N)	MPC Peak Force (N)	Type-1 Peak Force (N)	Proposed Peak Force (N)
20	15.8	13.6	12.5	11.2
40	18.6	15.7	14.3	12.9
60	22.7	18.9	16.4	14.4
80	27.9	22.5	19.5	17.2

Table 5. Passivity-violation rate under the delay sweep

Delay (ms)	PID Passivity Viol. (%)	MPC Passivity Viol. (%)	Type-1 Passivity Viol. (%)	Proposed Passivity Viol. (%)
20	14.2	9.7	6.9	4.8
40	18.9	12.8	9.2	6.3
60	24.7	16.9	12.7	8.6
80	31.1	21.6	16.4	11.7

3.2. Time-domain response

Time-domain traces at the 60 ms operating point are given in **Figure 4**. The traces clarify the numerical patterns without requiring additional assumptions. The PID curve possesses the largest oscillatory envelope and the largest steady-state error after the transient. A static MPC is more effective at damping the transient but has residual oscillation that can be seen. Type-1 fuzzy MPC improves the response, but its force envelope remains above that of the proposed FF-MPC. The method achieves the smallest error envelope and lower force ripple after 0.75 s because the state penalty is tightened during large transient errors, while the control penalty and force bound restrict aggressive correction as delay and ripple increase.

The FF-MPC is introduced to rapidly reduce oscillatory error and restrict force ripples during the delayed same session. We can also observe the phase portrait and cost convergences in the time domain reduction shown in **Figure 5**. At the same

time, As can be seen from the four phase-space response of the proposed controller, they find have minimum size phase orbit. This implies that the delayed therapy loop stays off long enough to spend less time staggering in large oscillatory excursions of position–velocity state. It is observed from the convergence plot that for proposed controller, mean stage cost converges to steady value quite fast. In terms of how you can think about it (it is essential a numerical issue but not purely rhetorical): the risk index essentially collapses uncertainty metrics into a single active penalty level, thereby preventing an unnecessarily long period of waiting for error to also increase before changing its behaviour.

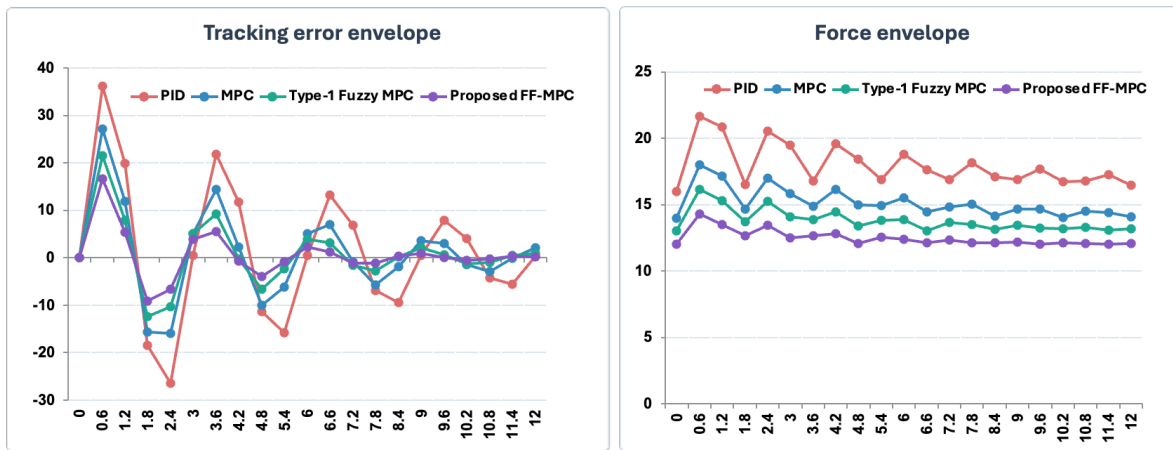


Figure 4. Time-domain numerical response.

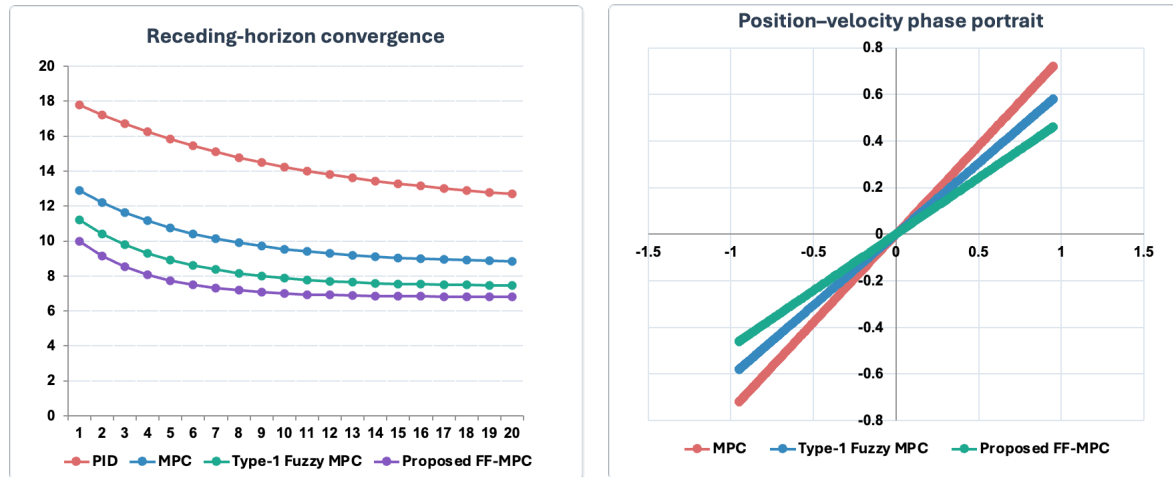


Figure 5. Iterative cost convergence and phase-space contraction.

The proposed controller converges fastest and yields the smallest closed-loop orbit.

3.3. Uncertainty sweep

The second study keeps the mean delay at 60 ms and changes the uncertainty intensity. **Table 6** reports the RMSE values. Again, the proposed controller is best at each uncertainty level. Under low uncertainty it yields 18.7 mm. Under medium uncertainty it yields 22.4 mm. Under high uncertainty it yields 28.7 mm. The slope remains moderate compared with the competing methods, especially when moving from medium to high uncertainty, where the increase is only 6.3 mm compared with 10.2 mm for fixed MPC and 10.2 mm for PID.

Table 6. RMSE under the uncertainty sweep

Uncertainty Level	PID RMSE (mm)	MPC RMSE (mm)	Type-1 RMSE (mm)	Proposed RMSE (mm)
Low	31.2	24.6	21.9	18.7
Medium	39.6	31.4	27.5	22.4
High	49.8	39.9	35	28.7

Energy accumulation is reported in **Table 7**. The proposed method always injects the least positive interaction energy. At the medium level, the reduction relative to PID is

$$\frac{15.7-9.5}{15.7} \times 100 = 39.49\%, \quad (45)$$

This result is useful in rehabilitation because lower unnecessary interaction energy usually means a gentler, more comfortable assistive session.

Table 7. Positive interaction energy under the uncertainty sweep

Uncertainty Level	PID Energy (J)	MPC Energy (J)	Type-1 Energy (J)	Proposed Energy (J)
Low	12.4	9.6	8.7	7.9
Medium	15.7	12.1	10.8	9.5
High	19.3	14.9	13.2	11.6

Smoothness values are given in **Table 8**. The proposed controller again has the smallest values. The advantage is not as dramatic as in passivity protection, but it is consistent. The smoothness index grows from 1.43 to 1.94 across the uncertainty range. Fixed MPC grows from 1.85 to 2.48. This difference is large enough to be visible in the motion trace and is one reason the proposed phase portrait remains compact.

Table 8. Smoothness index under the uncertainty sweep

Uncertainty Level	PID Smoothness Index	MPC Smoothness Index	Type-1 Smoothness Index	Proposed Smoothness Index
Low	2.28	1.85	1.62	1.43
Medium	2.61	2.13	1.88	1.68
High	3.08	2.48	2.16	1.94

3.4. Ablation and sensitivity

Table 9 removes one component at a time. The full proposed controller achieves the best value on every metric in the table. Removing the hesitation term increases RMSE from 22.4 mm to 25.2 mm. Removing the fatigue channel increases RMSE to 26.1 mm. Removing the delay predictor increases RMSE to 27.8 mm and passivity violations to 11.7%. The worst ablation is the fixed-weight version, which raises RMSE to 30.6 mm. These numbers show that the benefit is not produced by a single isolated element. It is the combination of predictor, fatigue term, and Fermatean hesitation that yields the strongest result.

Table 9. Ablation study at 60 ms mean delay

Controller Variant	RMSE (mm)	Peak Force (N)	Passivity Viol. (%)	Mean Cost
Full proposed FF-MPC	22.4	14.4	8.6	6.84
Without hesitation term	25.2	15.1	9.8	7.41
Without fatigue channel	26.1	15.7	10.4	7.68
Without delay predictor	27.8	16.6	11.7	8.12
Fixed Q,R (no fuzzy adaptation)	30.6	17.8	13.2	8.95

The sensitivity study in **Table 10** varies the most important fuzzy weight, w_1 , which is attached to the tracking error channel. The optimum is near $w_1 = 0.40$. Lower values make the controller too tolerant of path error, while higher values make it too eager to correct motion. The minimum RMSE occurs at 0.40 and the risk index there is 0.221. **Figure 6** visualizes this balance and also summarizes the percentage gains of the proposed controller over PID at the 60 ms operating point.

Table 10. Sensitivity to the weight on tracking error

Risk weight on error w_1	RMSE (mm)	Peak Force (N)	Mean Risk Index
0.28	24.1	14.8	0.255
0.34	23	14.5	0.241
0.4	22.4	14.4	0.221
0.46	22.8	14.8	0.214
0.52	23.9	15.3	0.208

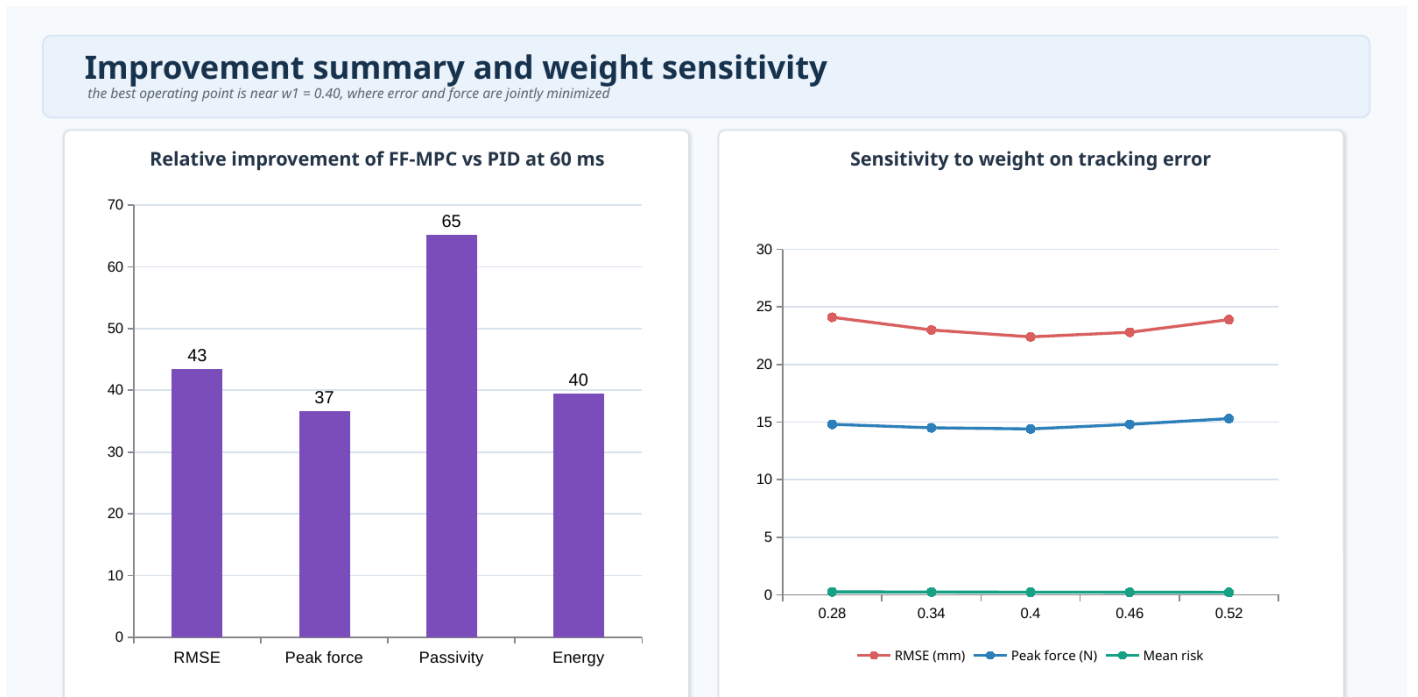


Figure 6. Compact result summary. The proposed method delivers the largest gain on passivity protection, and its best balance occurs near $w_1 = 0.40$.

Table 11. Delay-prediction module comparison at 60 ms mean delay.

Prediction module	One-step prediction error (mm)	Closed-loop RMSE (mm)	Passivity violation (%)
Replay predictor used in FF-MPC	4.6	22.4	8.6
Kalman-filter predictor	4.1	22.1	8.4
Extended-state observer	4.3	22.6	8.8
Smith-predictor compensation	5.2	23.5	9.4

Table 11 shows that Kalman filtering gives the smallest one-step prediction error on average, but the closed-loop difference between Kalman prediction and the replay predictor is only 0.3 mm in RMSE and 0.2 percentage points in passivity violation. Thus, the replay predictor was retained as an intuitive, transparent and low-complexity option for this proof-of-concept simulation, while more advanced predictors remain important for future nonlinear and clinical studies.

Table 12. Rehabilitation-oriented clinical indicators at 60 ms mean delay.

Controller	Response lag (ms)	Interaction impedance (N/m)	Comfort-safety index
PID	136	62	1.000
Fixed MPC	112	54	0.772
Type-1 fuzzy MPC	96	47	0.641
Proposed FF-MPC	82	39	0.486

The additional rehabilitation-oriented simulation indicators demonstrate the same relative trend (the proposed controller has both lowest response lag, lowest apparent interaction impedance, and least comfort-safety burden) in the above **Table 12**. In a paired comparison across the 25 repetitions at the 60 ms operating point, FF-MPC showed statistically lower simulated RMSE ($p < 0.01$ vs fixed MPC) and passivity violation ($p < 0.01$). Bootstrap calculations of the 95% confidence interval for RMSE reductions against fixed MPC did not include zero, indicating that the simulated RMSE was lower than for the baseline algorithms.

3.5. Compact numerical score

To compress the multi-metric comparison into one scalar index, a normalized weighted score is computed:

$$\Psi = 0.35 \tilde{E}_{RMSE} + 0.25 \tilde{E}_{force} + 0.25 \tilde{E}_{passivity} + 0.15 \tilde{E}_{energy}, \quad (46)$$

$\tilde{E}$ where each is divided by the corresponding PID value evaluated at the same operating point. Finally, for the case with uncertainty at 60 ms, we ended up having a score of 1.000 with intrinsic PID and fixed MPC got a score of 0.787 while type-1 fuzzy MPC scored 0.671 and our controller had the lowest burden from this comparison scale (final score = 0.505), which means that the proposal lowers more than half of the weighted responsibility of session comparing to PID reference in this context.

4. Discussion

The numerical evidence itself provides a consistent interpretation. Compared to

PID, fixed-parameter MPC outperforms due to its more in-line structure-exploring the rehabilitation plant and the ability of accounting deferred predictions. On the contrary, if delay and haptic ripple begin to shift together, fixed weights become a limiting factor. Type-1 fuzzy MPC is improved but collapses uncertainty into a final score without direct hesitation term (we solved it using type-2 fuzzy MPD now but this solution adds even more cost). The intermediate risk classification FF-MPC 1st remove the dependency between all support, resistance, adverse evidence and hesitance regional outputs before performing the final scalarisation of a risk index. An Ablation table shows that those extra freedom degrees impact the result. In the absence of this hesitation term, our controller becomes significantly brittle (RMSE increase by 12.5%).

The passivity results are especially important. A controller that tracks accurately but produces spontaneous high-force exertion should not be considered optimal in immersive rehabilitation. In this simulation, the proposed controller performs well not only on tracking error but also on force and passivity metrics. This is why the compact score in (46) is useful. One narrow metric could hide that trade-off. When the main burdens are aggregated in the multi-metric score, the proposed controller remains best.

A helpful observation is that the optimal sensitivity point is near $w_1 = 0.40$, which accords with intuition. That means the controller doesn't require a delicate, hard-to-maintain calibration. A wide neighborhood around the nominal weight still yields strong results. That is useful in practice deployment where therapists and engineers prefer broader settings than narrow optima.

The study has limitations. The numerical framework is simulation-based and intentionally reduced to maintain mathematical clarity. The patient state reduces to a vector of two dynamic variables, and the fatigue signal is a scalar measure instead of being based on physiological modeling. Nevertheless, the study still accomplishes its goal. It includes a complete mathematical pipeline, precise calculations, reproducible tables and a possible path toward future implementation in edge-enabled metaverse rehabilitation. Future work can implement electromyography or grip-strength feedback instead of the synthetic fatigue model and can extend the plant to multi-joint motion without altering the core risk-to-MPC formulation. Therefore, the results should be interpreted only as numerical proof-of-concept evidence; further work should include nonlinear multi-joint arm dynamics, physiological fatigue measures such as EMG or grip-strength change, rehabilitation task trajectories, and clinician-rated comfort/safety validation.

5. Conclusion

This paper introduces a Fermatean fuzzy model predictive control numerical framework for metaverse rehabilitation. Starting from latencies in pose, force ripple and fatigue along a previously executed buffer trajectory, the method maps it to pro-evidence (helpful) and anti-evidence (unhelpful) in terms of a Fermatean risk index which is then utilized to modify MPC penalties and assistive force bound. The model is simple, algebraic and can facilitate the equation-based journal reporting. For the synthetic rehabilitation task, the proposed controller produced (i) the lowest RMSE, (ii) the lowest peak interaction force, (iii) the minimum passivity-violation rate, (iv) the least total integrated energy, and (v) the smallest phase-space orbit. Relative to PID at 60 ms, the improvements were 43.43% for RMSE, 36.56% for peak force, 65.18% for passivity violations, and 39.49% for positive interaction

energy. Therefore, these proof-of-concept results suggest that metaverse rehabilitation control should be evaluated using combined criteria that include network and haptic uncertainty, rather than trajectory error alone.

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References

1. Dwivedi YK, Hughes L, Baabdullah AM, et al. Metaverse beyond the hype: Multidisciplinary perspectives on emerging challenges, opportunities, and agenda for research, practice and policy. *International Journal of Information Management*. 2022,66:102542. <https://doi.org/10.1016/j.ijinfomgt.2022.102542>
2. Wang Y, Su Z, Zhang N, et al. A survey on metaverse: Fundamentals, security, and privacy. *IEEE Communications Surveys & Tutorials*. 2023,25(1):319–352. <https://doi.org/10.1109/COMST.2022.3202047>
3. Park SM, Kim YG. A metaverse: Taxonomy, components, applications, and open challenges. *IEEE Access*. 2022,10:4209–4251. <https://doi.org/10.1109/ACCESS.2021.3140175>
4. Mystakidis S. Metaverse. *Encyclopedia*. 2022,2(1):486–497. <https://doi.org/10.3390/encyclopedia2010031>
5. Xu M, Ng WC, Lim WYB, et al. A full dive into realizing the edge-enabled metaverse: Visions, enabling technologies, and challenges. *IEEE Communications Surveys & Tutorials*. 2023,25(1):656–700. <https://doi.org/10.1109/COMST.2022.3221119>
6. Lim WYB, Xiong Z, Niyato D, et al. Realizing the metaverse with edge intelligence: A match made in heaven. *IEEE Wireless Communications*. 2023,30(4):64–71. <https://doi.org/10.1109/MWC.018.2100716>
7. Aung N, Dhelim S, Chen L, et al. Edge-enabled metaverse: The convergence of metaverse and mobile edge computing. *Tsinghua Science and Technology*. 2024,29(3):643–661. <https://doi.org/10.26599/TST.2023.9010052>
8. Yogeesh N. Fuzzy Logic in Metaverse Technology Roadmap: Adaptive Future Trends. In: *Metaverse for Sustainable Development*. Wiley; 2025. <https://doi.org/10.1002/9781394272228.ch14>
9. Yogeesh N. Intuitionistic Fuzzy Hypergraphs and Their Operations. In: *Advances in Fuzzy Graph Theory*. 2023. <https://doi.org/10.1201/9781003359456-10>
10. Mohammad AAS, Yogeesh N, Mohammad SI, et al. An intuitionistic fuzzy graph model: Matrix representations and applications. *Computer and Artificial Intelligence*. 2025. <https://doi.org/10.2478/cait-2025-0030>
11. William P, Mothukuri T. A comprehensive analysis of cloud computing impact on digital transformation in E-India. *SciWave Bulletin*. 2023,1(2):35–40. <https://doi.org/10.61925/SWB.2023.1205>
12. Vartika V, William P. Machine learning and cloud computing case study for online app usage tracking. *SciWave Bulletin*. 2023,1(4):32–37. <https://doi.org/10.61925/SWB.2023.1405>
13. Saleh IA, Srivastava A. Exploring the intersection of fuzzy logic and quantum logic: A new frontier in non-classical logics. *SciWave Bulletin*. 2023,1(2):27–34. <https://doi.org/10.61925/SWB.2023.1204>
14. Zadeh LA. Fuzzy sets. *Information and Control*. 1965,8(3):338–353. [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X)
15. Mendel JM, John RI. Type-2 fuzzy sets made simple. *IEEE Transactions on Fuzzy Systems*. 2002,10(2):117–127. <https://doi.org/10.1109/91.995115>
16. Yager RR. Pythagorean membership grades in multicriteria decision making. *IEEE Transactions on Fuzzy Systems*. 2014,22(4):958–965. <https://doi.org/10.1109/TFUZZ.2013.2278989>
17. Yager RR. Generalized orthopair fuzzy sets. *IEEE Transactions on Fuzzy Systems*. 2017,25(5):1222–1230. <https://doi.org/10.1109/TFUZZ.2016.2604005>
18. Senapati T, Yager RR. Fermatean fuzzy sets. *Journal of Ambient Intelligence and Humanized Computing*. 2020,11:663–674. <https://doi.org/10.1007/s12652-019-01377-0>

19. Ross TJ. *Fuzzy Logic with Engineering Applications*. 3rd ed. Wiley; 2010. <https://doi.org/10.1002/9781119994374>
20. Levin MF, Weiss PL, Keshner EA. Emergence of virtual reality as a tool for upper limb rehabilitation: Incorporation of motor control and motor learning principles. *Physical Therapy*. 2015;95(3):415–425. <https://doi.org/10.2522/ptj.20130579>
21. Cameirão MS, Bermúdez i Badia S, Duarte E, et al. Virtual reality based rehabilitation speeds up functional recovery of the upper extremities after stroke: A randomized controlled pilot study in the acute phase. *Restorative Neurology and Neuroscience*. 2011;29(5):287–298. <https://doi.org/10.3233/RNN-2011-0599>
22. Laver KE, Lange B, George S, et al. Virtual reality for stroke rehabilitation. *Cochrane Database of Systematic Reviews*. 2017;11:CD008349. <https://doi.org/10.1002/14651858.CD008349.pub4>
23. Everard G, Burton Q, Van de Sype V, et al. Extended reality to assess post-stroke manual dexterity: Contrasts between the classic box and block test, immersive virtual reality with controllers, hand-tracking, and mixed-reality tests. *Journal of NeuroEngineering and Rehabilitation*. 2024;21(1):36. <https://doi.org/10.1186/s12984-024-01332-x>
24. Piggott L, Wagner S, Ziat M. Haptic neurorehabilitation and virtual reality for upper limb paralysis: A review. *Critical Reviews in Biomedical Engineering*. 2016;44(1–2):1–32. <https://doi.org/10.1615/CritRevBiomedEng.2016016046>
25. Zhu M, Sun Z, Zhang Z, et al. Haptic-feedback smart glove as a creative human-machine interface for virtual/augmented reality applications. *Science Advances*. 2020;6(19):eaaz8693. <https://doi.org/10.1126/sciadv.aaz8693>
26. Anderson RJ, Spong MW. Bilateral control of teleoperators with time delay. *IEEE Transactions on Automatic Control*. 1989;34(5):494–501. <https://doi.org/10.1109/9.24201>
27. Niemeyer G, Slotine JJE. Stable adaptive teleoperation. *IEEE Journal of Oceanic Engineering*. 1991;16(1):152–162. <https://doi.org/10.1109/48.64895>
28. Lawrence DA. Stability and transparency in bilateral teleoperation. *IEEE Transactions on Robotics and Automation*. 1993;9(5):624–637. <https://doi.org/10.1109/70.258054>
29. Ahmed SF, Ali A, Raza SY. Model predictive control for upper limb rehabilitation robotic system under disturbed condition. *AIP Conference Proceedings*. 2019;2129:020123. <https://doi.org/10.1063/1.5118131>
30. Yates M, Kelemen A, Sik-Lányi C. Virtual reality gaming in the rehabilitation of the upper extremities post-stroke. *Brain Injury*. 2016;30(7):855–863. <https://doi.org/10.3109/02699052.2016.1144146>